DEPARTMENT OF EDUCATION

GRADE 9

MATHEMATICS

UNIT 6

DESIGN IN 2D AND 3D GEOMETRY (2)

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Acknowledgements

We acknowledge the contribution of all Secondary Teachers who in one way or another helped to develop this Course.

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SECRETARY'S MESSAGE

Achieving a better future by individuals students, their families, communities or the nation as a whole, depends on the curriculum and the way it is delivered.

This course is part and parcel of the new reformed curriculum – the Outcome Base Education (OBE). Its learning outcomes are student centred and written in terms that allow them to be demonstrated, assessed and measured.

It maintains the rationale, goals, aims and principles of the national OBE curriculum and identifies the knowledge, skills, attitudes and values that students should achieve.

This is a provision of Flexible, Open and Distance Education as a alternative pathway of formal education.

The Course promotes Papua New Guinea values and beliefs which are found in our constitution, Government policies and reports. It is developed in line with the National Education Plan (2005 – 2014) and addresses an increase in the number of school leavers which has been coupled with a limited of access to secondary and higher educational institutions.

Flexible, Open and Distance Education is guided by the Department of Education's Mission which is fivefold;

- to facilitate and promote integral development of every individual
- to develop and encourage an education system which satisfies the requirements of Papua New Guinea and its people
- to establish, preserve, and improve standards of education throughout Papua New Guinea
- to make the benefits of such education available as widely as possible to all of the people
- to make education accessible to the physically, mentally and socially handicapped as well as to those who are educationally disadvantaged.

The College is enhanced to provide alternative and comparable path ways for students and adults to complete their education, through one system, many path ways and same learning outcomes.

It is our vision that Papua New Guineans harness all appropriate and affordable technologies to pursue this program.

I commend all those teachers, curriculum writers and instructional designers, who have contributed so much in developing this course.

……………………………………

DR. UKE KOMBRA, PhD
Acting Secretary for Education

UNIT 6: DESIGN IN 2D AND 3D GEOMETRY (2)

Dear Student

This is the sixth Unit of the Grade 9 Mathematics Course. This unit is based on the NDOE Lower Secondary Syllabus and Curriculum Framework for Grade 9.

This Unit consists of four topics:

Topic 1: Transformation and Symmetry
Topic 2: Similarity and Congruence
Topic 3: Circles
Topic 4: Construction

In Topic 1-**Transformation and Symmetry**-You will revise concepts in symmetry and types of symmetry. You will also learn the meaning of transformation and its types such as rotation, reflection, translation and enlargement. Lastly, you will solve problems involving scale drawings.

In Topic 2- **Similarity and Congruence**- You will define and identify the properties of similar and congruent figures. You will also learn to solve problems involving similar figures as well as finding the areas of similar triangles.

In Topic 3- **Circles**- You will learn and revise the meaning of a circle and its parts. You will also learn to find the circumference and arc length of a circle as well as finding the area of a circle, sector of a circle and segment of a circle using formulas and rules.

In Topic 4- **Constructions**- You will learn the different methods of drawing solid objects like isometric and perspective drawings. You will also learn to draw triangles, line bisector, angle bisector and regular shapes using a pair of compass and a pencil. And lastly, you will learn to construct regular shapes using their nets.

You will find that each lesson has reading material to study, worked examples and a Practice Exercise. The answers to the practice exercise are given at the end of each Topic.

All the lessons are written in simple language with comic characters to guide you. The practice exercises are graded to help you to learn the process of working out problems.

We hope you enjoy learning this Unit.

Mathematics Department
FODE

STUDY GUIDE

Follow the steps given below as you work through the Unit.

Step 1: Start with TOPIC 1 Lesson 1 and work through it.
Step 2: When you complete Lesson 1, do Practice Exercise 1.
Step 3: After you have completed Practice Exercise 1, check your work. The answers are given at the end of the TOPIC 1.
Step 4: Then, revise Lesson 1 and correct your mistakes, if any.
Step 5: When you have completed all these steps, tick the check-box for the Lesson, on the Contents Page (page 3) like this:

√ Lesson 1: Transformation

Then go on to the next Lesson. Repeat the same process until you complete all of the lessons in Topic 1.

> As you complete each lesson, tick the check-box for that lesson, on the Content Page 3, like this √ .This helps you to check on your progress.

Step 6: Revise the Topic using Topic 1 Summary, then, do Topic test 1 in Assignment 6.

Then go on to the next Topic. Repeat the same process until you complete all of the four Topics in Unit 6.

Assignment: (Four Topics and a Unit Test)
When you have revised each Topic using the Topic Summary, do the Topic Test in your Assignment. The Unit book tells you when to do each Topic Test.

When you have completed the four Sub-strand Tests, revise well and do the Strand test. The Assignment tells you when to do the Strand Test.

> The Topic Tests and the Unit test in the Assignment will be marked by your Distance Teacher. The marks you score in each Assignment will count towards your final mark. If you score less than 50%, you will repeat that Assignment.

Remember, if you score less than 50% in three Assignments, you will not be allowed to continue. So, work carefully and make sure that you pass all of the Assignments.

TOPIC 1

TRANSFORMATION AND SYMMETRY

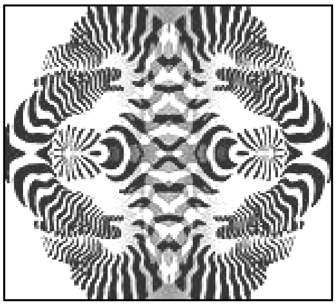
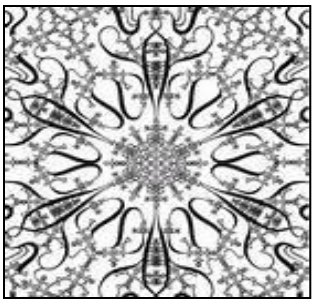
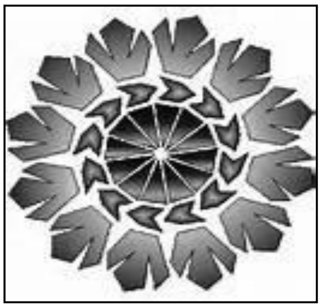
TOPIC 1: TRANSFORMATION AND SYMMETRY

Introduction

The world around us provides many beautiful examples of transformations and symmetries. When you are on an amusement park ride, you are undergoing a transformation. Ferris wheels, merry-go-rounds, free-fall rides and water slides are examples of **transformations**. The wings of butterflies and the petals of a flower are only some examples of **symmetry** in nature.

We see examples of symmetry and transformation in almost everything around us - from the structure of a butterfly, of a fern, of a flower to the pattern of a patchwork quilt.



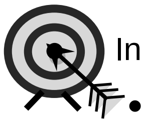





Symmetry is the preservation of form and configuration across a point, a line, or a plane. In informal terms, symmetry is the ability to take a shape and match it exactly to another shape. The techniques that are used to "take a shape and match it exactly to another" are called **transformations** and include **translations**, **reflections**, **rotations**, and **glide reflections**.

This Topic will provide you the opportunities to use mathematics creatively by working and exploring the concepts of transformation and symmetry and learning to create designs using the step by step instructions, techniques and skills to be learnt.

Lesson 1: Transformation

You have learnt something about transformation in Strand 2 of Grade 8 Mathematics.



In this lesson you will:

- define what transformation is
- identify the main types of transformation
- describe translations, rotations, reflections, and. enlargements
- transform an object using translation, reflection, reduction, rotation and enlargement.

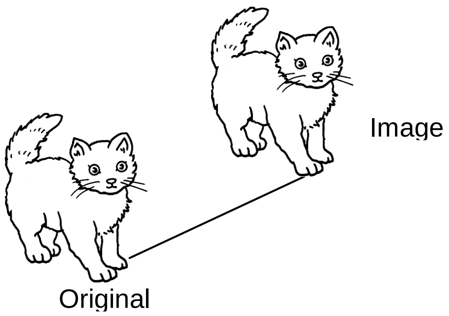
A point, line or a figure in a plane can be moved to a new position in that plane. Such movement is called a **transformation**.

Transformation is a change in position, size or form of a shape.

Transformation is a process which enables us to begin with any given figure and obtain a corresponding figure in the same plane. The object or figure you start with is called the ***original*** and the object or figure into which it is transformed is called the **image**.

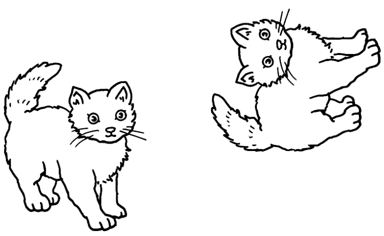
There are four main types of transformation. Each of the type changes (moves) the position of an object or figure.

1. Sliding Transformation (Translation)



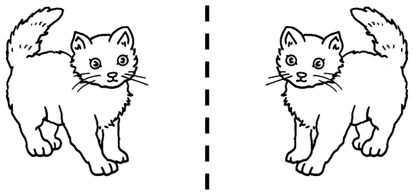
The original object is moved by sliding. Every point moved the same distance and in the same direction.

2. Spin Transformation (Rotation)



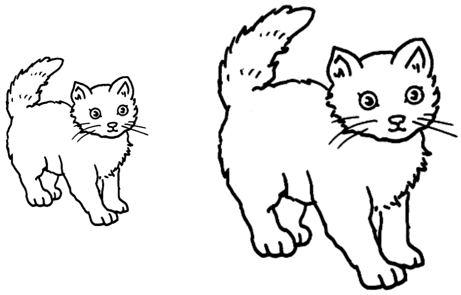
The original object is moved by spinning or turning it about a point.

3. Turnover Transformation (Reflection)



The original object is changed into its mirror image. The object and its image are oppositely congruent (i.e. one can be fitted exactly on top of the other after being "turned or flipped" over.

4. Size Transformation(Enlargement or Reduction)



The original object is changed in size but its shape stays the same.

Now let us look at each type one by one.

Translation

This is the correct name for the sliding transformation. We obtain the image by sliding the figure (without rotation or reflection, a certain given distance at a given direction.

A translation moves every points in a shape the same distance and the same direction.

For example, every point on the flag in Figure 1 below is moved by 3 units in the **x** direction and 1 unit in the **y** direction.

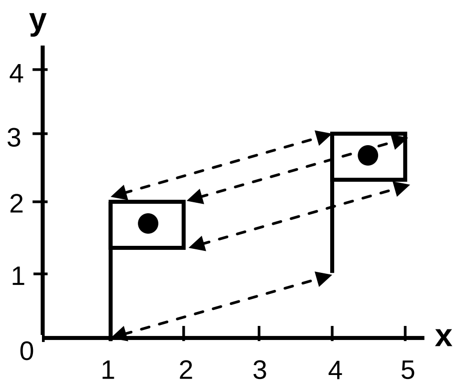


FIGURE 1

Notice that each of the dashed arrows is drawn from an original point to its image. We can show the direction and distance of any translation by drawing any such translation arrow, which is called "**vector**". The word "**vector**" means "**carrier**", and shows the direction and distance we must "**carry**" each original point to its image.

To specify a translation we must state the direction and distance. We may show the direction and distance by drawing a "vector" which points out the direction and whose length gives the distance.

We can write this translation using the **vector notation:** $\begin{pmatrix} \pm \mathbf{x} \\ \pm \mathbf{y} \end{pmatrix}$

The horizontal movement (x direction) is always written at the top.

The **plus (+) signs** indicate or show that the movement is to the **right** or **up**.

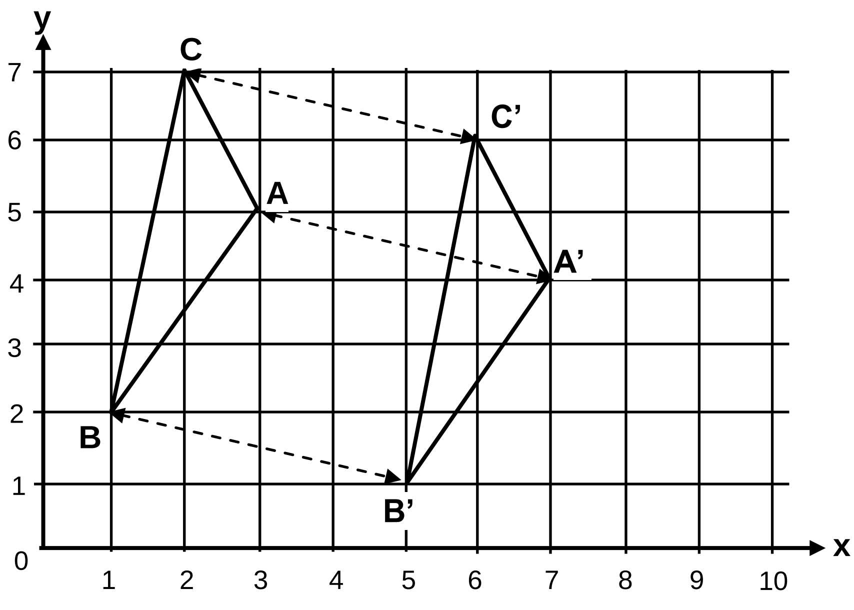
The **minus (-) signs** indicate or show that the movement is to the **left** or **down**.

So, for the translation of the flag in FIGURE 1, we write it like this: $\begin{pmatrix} +\mathbf{3} \\ +\mathbf{1} \end{pmatrix}$

Here is example on how to perform a translation.

Example 1

In the Triangle ABC, the Point A is (3, 5), Point B is (1, 2) and Point C is (2, 7). Find the triangle after a translation $\begin{pmatrix} +4 \\ -1 \end{pmatrix}$.



Solution:

The image is now Triangle A'B'C'.

You can find the position of Point A' by writing the coordinates of Point A as $\begin{pmatrix} 3 \\ 5 \end{pmatrix}$ and adding the translation $\begin{pmatrix} +4 \\ -1 \end{pmatrix}$.

Hence, $\begin{pmatrix}3\\5\end{pmatrix} + \begin{pmatrix}+4\\-1\end{pmatrix} = \begin{pmatrix}3+4\\5-1\end{pmatrix} = \begin{pmatrix}7\\4\end{pmatrix}$. Point A' is the point (7, 4).

Similarly for B': $\begin{pmatrix}1\\2\end{pmatrix} + \begin{pmatrix}+4\\-1\end{pmatrix} = \begin{pmatrix}1+4\\2-1\end{pmatrix} = \begin{pmatrix}5\\1\end{pmatrix}$. Point B' is the point (5. 1)

And for C': $\begin{pmatrix}2\\7\end{pmatrix} + \begin{pmatrix}+4\\-1\end{pmatrix} = \begin{pmatrix}2+4\\7-1\end{pmatrix} = \begin{pmatrix}6\\6\end{pmatrix}$. Point B' is the point (6. 6).

Rotation

This is the correct name for the spin transformation. A rotation is a turning movement. In a rotation every point of the object or shape turns through the same angle about a point.

Describing a rotation requires three pieces of information.

(a) the angle of rotation
(b) the direction of the rotation(anticlockwise is positive)
(c) the centre of the rotation.

Example

Take a look at the shape in Figure 2.

(a) Rotating Shape A - 90º clockwise about Point O, gives the image B.

(b) Rotating Shape A -180º clockwise about Point O, gives the image C.

(c) Rotating Shape A -270º clockwise about Point O, gives the image D.

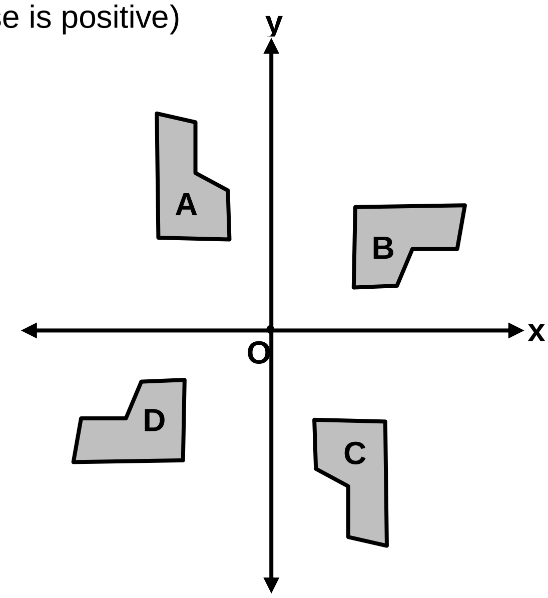


FIGURE 2

Here are some important properties of Rotation.

1. Rotation does not change the size and shape. The object or shape and its image are directly congruent
2. The centre of rotation is the only fixed point. All other points move.
3. The centre of rotation can be anywhere in the plane.
4. Every line turns through the same angle. This is the **angle of rotation**.

To rotate an object about a centre, rotate each vertex (corner point) through the given angle. Join up the image points to make the matching image shape.

You can find the angle of rotation given an object or shape and its image.

Here is how to do it.

If you are given the centre of rotation, then draw lines joining an object point and its image point to the centre. The angle between the lines is the angle of rotation. You can measure it. See FIGURE 3.

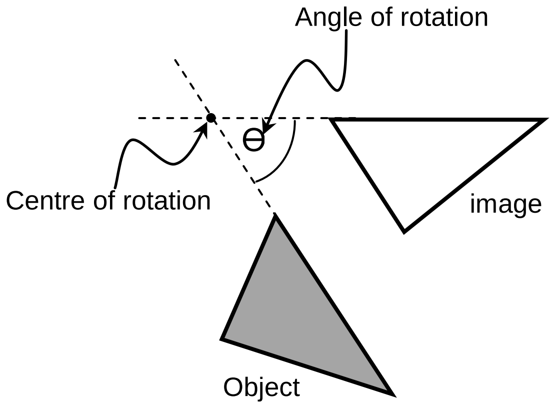


FIGURE 3

You can also find the centre of rotation given an object and its image.

Here is how to do it.

Steps

1. Join two matching points on the object and image.
2. Draw the perpendicular bisector of the line.
3. Repeat steps 1 and 2 for the two other matching points.

   The centre of rotation is where the two perpendicular bisectors meet or cross.

   See FIGURE 4.

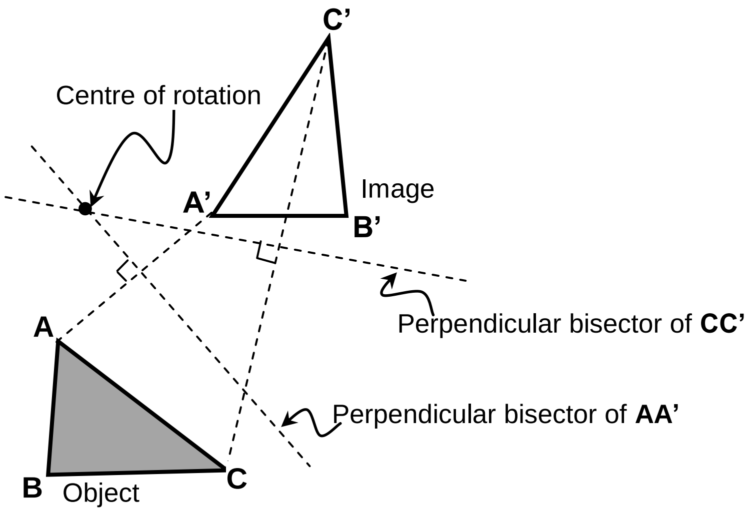


FIGURE 4

Remember:

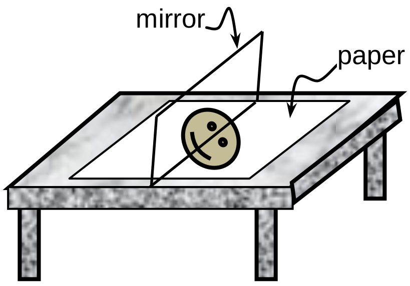
Original segments, angles and triangles are congruent to their images under rotation.

Reflection

This is the correct name for the turnover transformation. Reflection is the most interesting type of transformation. We now find why the name reflection is used.

First do the following activity.

1. Draw any figure on a sheet of paper and place it on the table. Place a mirror so that its edge rests on the paper and its plane is perpendicular to the plane of the paper as shown below.



Notice that the figure and its reflection or mirror image lie in the same plane. Every point, segment and angle of the figure has a corresponding mirror image.

In a reflection, an object is changed into its mirror image. To obtain or describe a reflection you must know the position of the **mirror line** or **axis of reflection**.

Here are some properties of reflection.

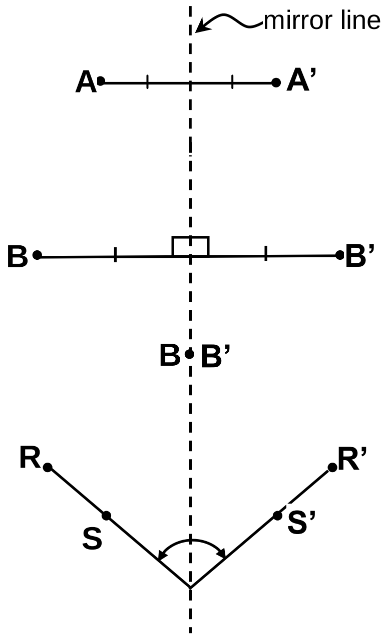
1. Reflection does not change size and shape. But it does “flip” the shape over. An object and its image are oppositely congruent (one can be fitted exactly on top of the image having flipped over)

2. A point and its image are the same distance from the mirror line they are on opposite sides of it.

3. A line joining the point to its image is perpendicular to the mirror line.It is also bisected by the mirror line.

4. The image of a point on the mirror line is the point itself.

5. The angle between a line and the mirror line is equal to the angle between its image and the mirror line. The mirror line bisects the angle between a line and its image.



To specify a reflection, we need an axis of reflection (mirror line).

When reflecting an object in a mirror line, reflect each vertex (corner point) in turn. Make each image point the same perpendicular distance from the mirror line as the object point, but on the opposite side.

Example

Use the grid below to reflect the shaded **L** shape in the mirror line.

Solution:

B $\longleftrightarrow$ B'

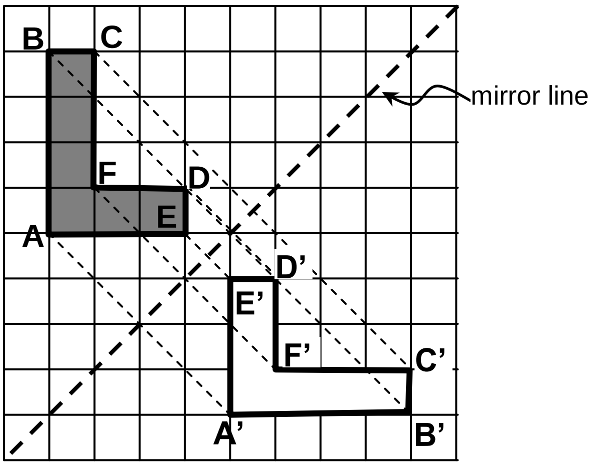
C $\longleftrightarrow$ C'

D $\longleftrightarrow$ D'

E $\longleftrightarrow$ E'

F $\longleftrightarrow$ F'

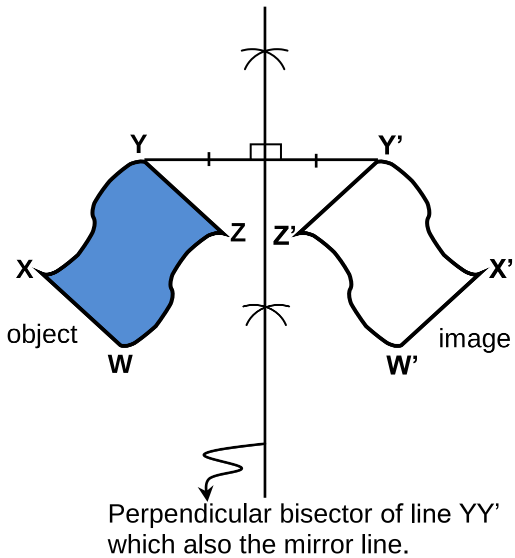
($\longleftrightarrow$) means "corresponds to" or "matches to"



Shape BCDEF is congruent to shape B'C'D'E'F'.

You can find the mirror line or axis of reflection given an object and its image. The steps are as follow:

1. Join two matching points on the object and its image.
2. Draw the perpendicular bisector of this line.
3. The perpendicular bisector is the mirror line or axis of reflection.



Enlargement

When you enlarge something you change its size. Its shape stays the same. So the original shape and its enlargement are similar. Matching lengths change in size. But matching angels stay the same.

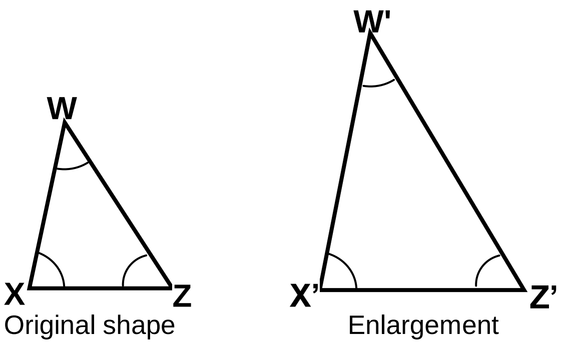


FIGURE 6

The object (starting original shape) and the image (enlarged shape) in FIGURE 6 have been labelled in a special way. The object is Triangle **WXZ**. The same letters are used for matching points on the image but with a dash (**'**) after each. So the image is Triangle **W'X'Z'**.

W $\longleftrightarrow$ **W'**

X $\longleftrightarrow$ **X'**

Z $\longleftrightarrow$ **Z'**

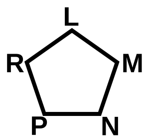
An object and its image are often labelled this way like what is also used for other previous transformations.

To carry out an enlargement requires two pieces of information.

1. The Scale factor (the number used as a multiplier in size transformation)
2. The centre of enlargement (the point around which the object is enlarged)

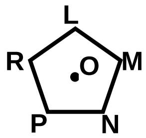
You can enlarge a shape using a centre of enlargement and a scale factor. This is called the **spider method or ray method**.

Follow these steps to draw an enlargement of the shape below using this method. Use a scale factor of 1.

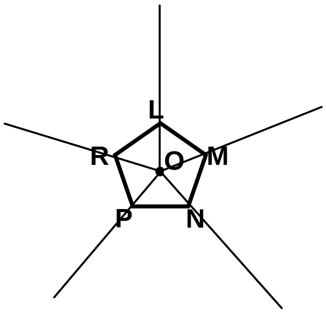


Steps

1. Mark the centre of enlargement O.



2. Draw a straight line from O through each vertex (corner point) of the shape in turn.

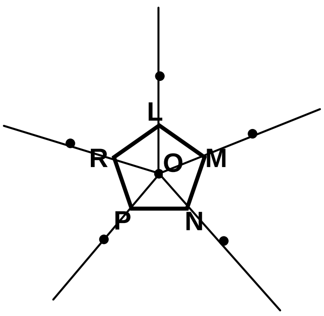


3. Measure the distance from O to each vertex. Put your measurement in a table.

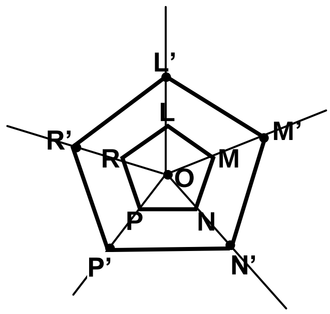
OBJECT		IMAGE	
O to L	0.6 cm	O to L’	1 x 0.6 cm = 0.6 cm
O to M	0.6 cm	O to M’	1 x 0.6 cm = 0.6 cm
O to N	0.6 cm	O to N’	1 x 0.6 cm = 0.6 cm
O to P	0.6 cm	O to P’	1 x 0.6 cm = 0.6 cm
O to R	0.6 cm	O to R’	1 x 0.6 cm = 0.6 cm

4. Multiply each length by the scale factor. Put your answer in the table too.

5. Used your enlarged length to mark the image of each vertex.



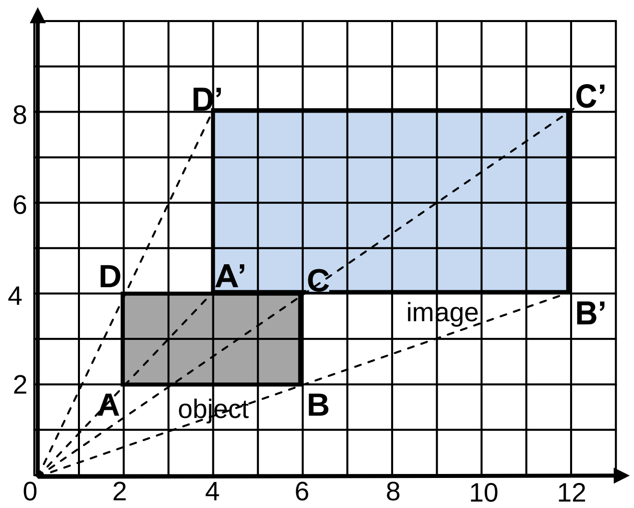
6. Join up the image points in the correct order to make the image shape.



The centre of enlargement does not have to be inside the object or shape. It can be outside or on the edge of the object or shape.

Example 1

Rectangle ABCD is enlarged by a scale factor of 2, the centre of enlargement is the origin.



This means that, OA' = 2 x OA measured along OA extended

OB' = 2 x OB measured along OB extended

OC' = 2 x OC measured along OC extended

OD' = 2 x OC measured along OD extended

The image A'B'C'D' is four times the size of the object.

Enlargement are often drawn on a grid. You may be given the coordinates of the centre of enlargement and the vertices of the object.

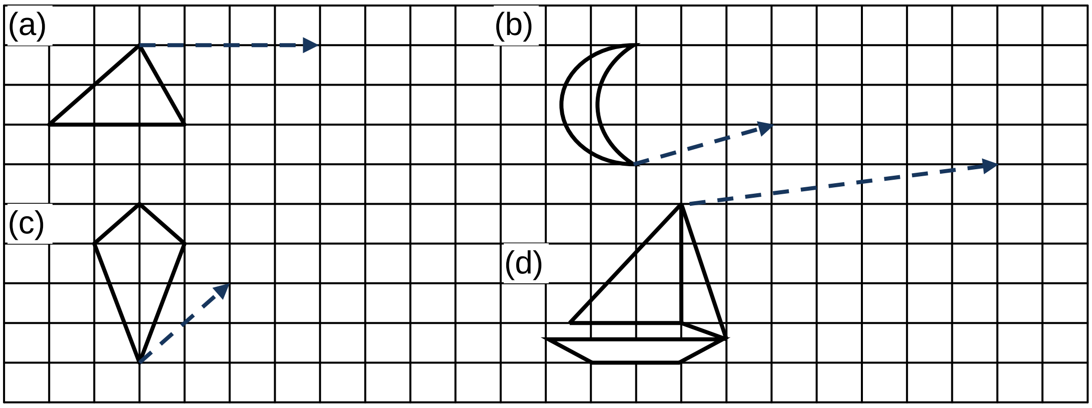
When the object or shape is enlarged, the size of each length is changed in the same way.

Enlargement changes the size of an object. The other three types of transformation do not.

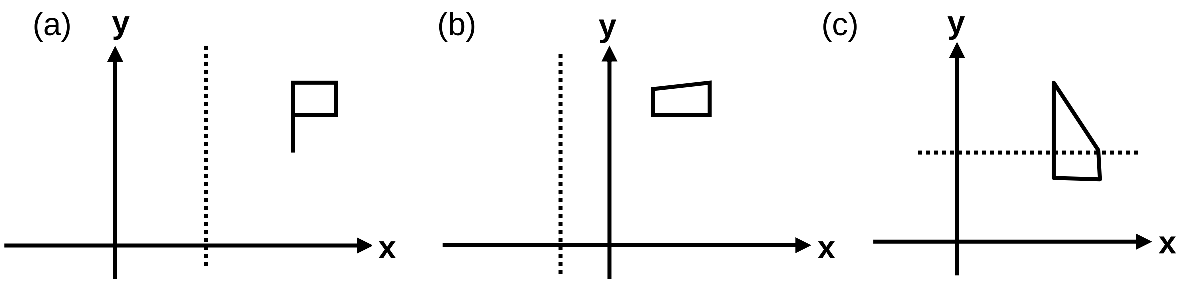
NOW DO PRACTICE EXERCISE 1

Practice Exercise 1

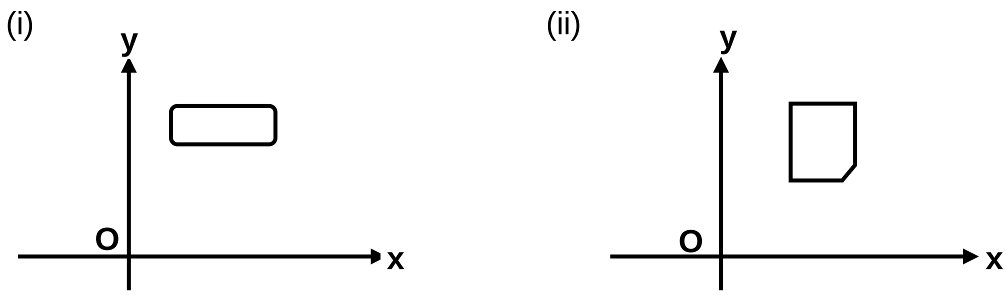
1. In each of the following diagrams a translation is specified by a dashed arrow. Copy the diagram on to the grid, and using the grid lines to help you, draw the image of the original figure.



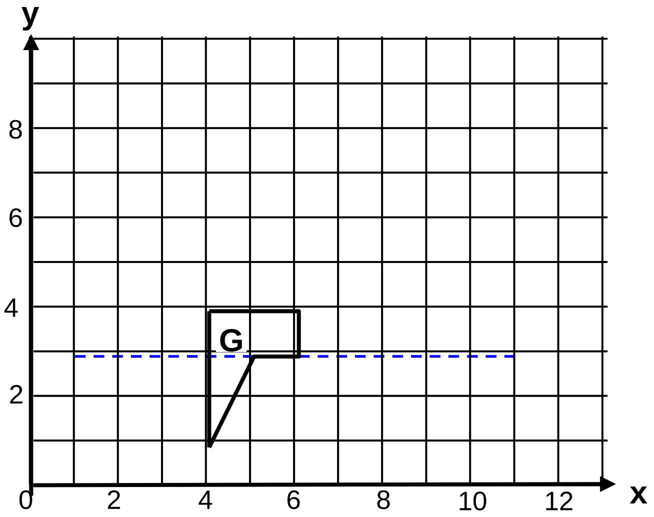
2. Draw the image of each of these shapes after a reflection in the mirror line, shown dotted.



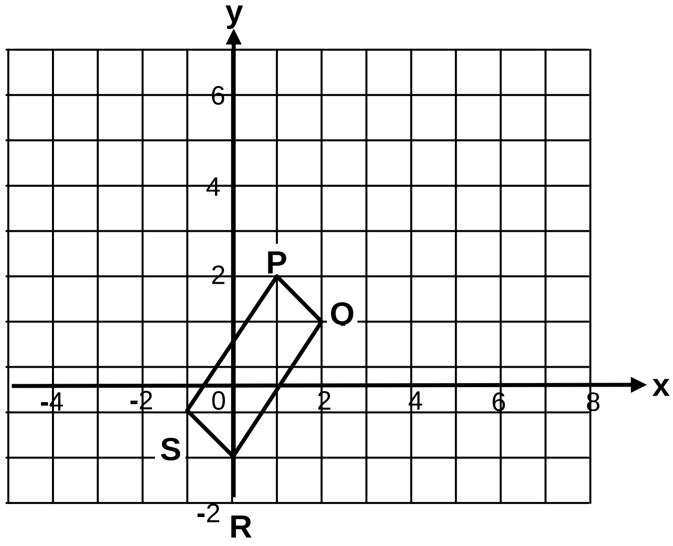
3. Give the image of these shapes after
(a) a rotation about O through 180º anticlockwise
(b) a rotation about O through 90º anti clockwise
(c) a rotation about O through 120º anti clockwise.



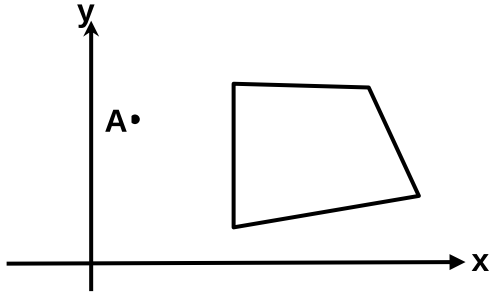
4. Find the image of shape G after an enlargement centre (1, 3), of scale factor 2.



5. The quadrilateral PQRS is reflected into a quadrilateral P'Q'R'S'. Draw the mirror line or axis of reflection.



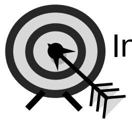
6. Draw the image of this shape using A as centre of enlargement and a scale factor of 3.



CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 1.

Lesson 2: Line Symmetry

You've learnt something about Symmetry in your Grade 7 and 8 Mathematics.



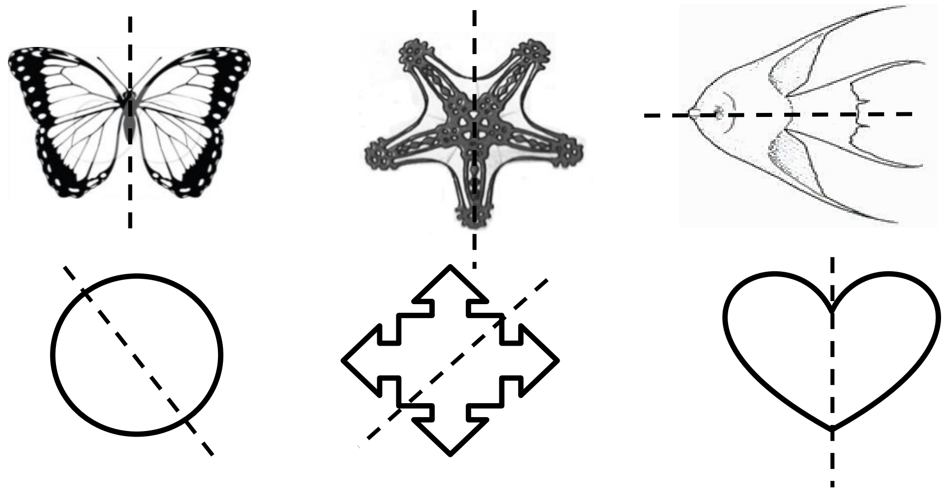
In this lesson, you will:

- revise the meaning of symmetry
- identify one or more lines of symmetry in 2D shapes
- recognize and create 2D shapes that have lines of symmetry

You can find symmetry in nature, architecture and in art. Symmetry exists all around us and many people see it as being a thing of beauty. Nature provides many beautiful examples of symmetry, such as the wings of the butterfly, starfish, or the petals of a flower.

Symmetrical shapes are shapes or objects that have two halves that are congruent (exactly the same size and shape).

Below are symmetrical shapes and figures.



These figures are symmetrical in relation to the dashed line. The line is called a **line or axis of symmetry**. This means that one half of the figure is the mirror image of the other part.

Line or Axis of symmetry is the line dividing the shape into two identical parts that are mirror images of one another.

Imagine that you folded the figures along the dashed line. Then both sides would exactly meet or place a mirror along the dashed line. You see the other half of the figure or shape is reflected in the mirror.

We say that the shape has **line symmetry or reflectional symmetry**. This is the simplest type of symmetry.

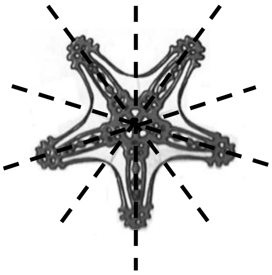
Example 1

The butterfly and the heart shaped figure, each has a **line symmetry** because one side of the dashed line is a mirror image of the other side.

Some shapes can be folded in different ways so that the sides meet.

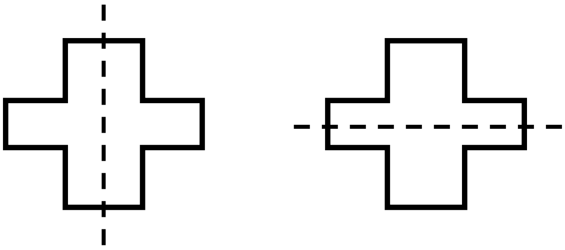
Example 2

Look at the starfish. It has five different lines or axes of symmetry.



Example 3

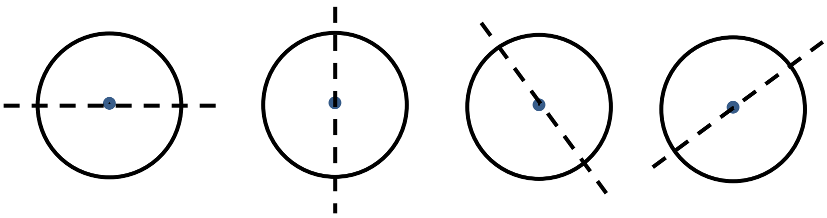
The cross shaped figure below has two different lines or axes of symmetry.



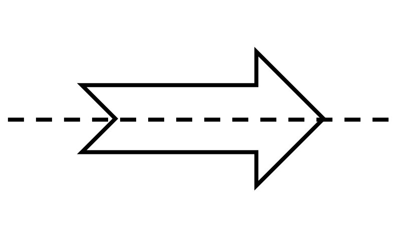
Example 4

A circle has infinitely many axes of symmetry because there is an infinite number of different ways to fold it and have all its points match.

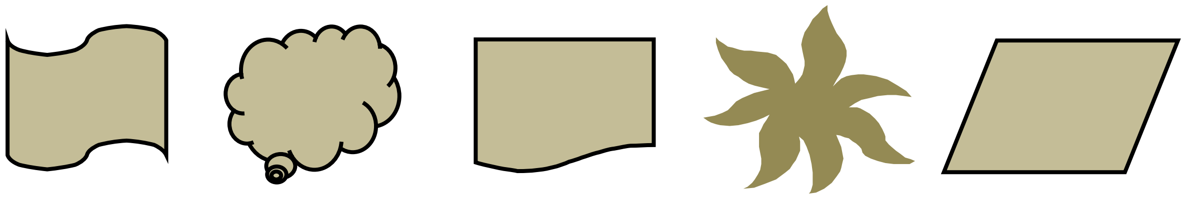
Any line that you can draw through the centre point of a circle is a line or axis of symmetry. So, we can't even count how many lines of symmetry a circle has. Have a look at some examples drawn for you below.



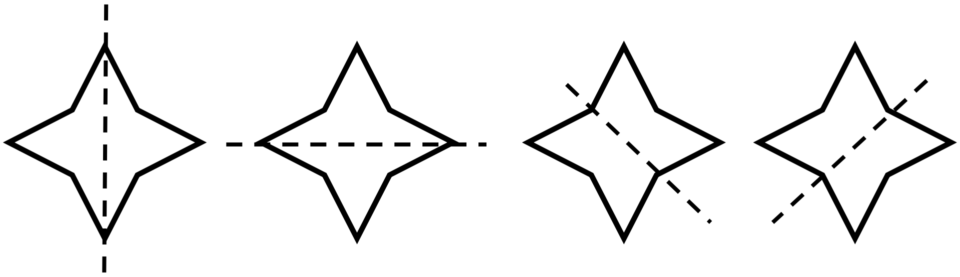
Some shapes have only one line or axis of symmetry like the ones below.



Many figures are not symmetrical at all.



Look again at the shape with axis of symmetry drawn.



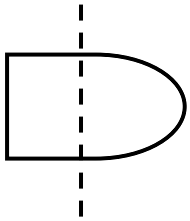
The shape is divided into two identical parts four times along the dashed lines. There are four dashed lines. The shape has four (4) axes of symmetry

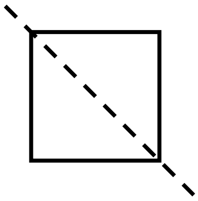
- **Symmetry means exact likeness in size and shape between the two sides of something.**
- **A shape or figure has line symmetry when one half exactly folds on to the other half.**
- **Axis of symmetry is the line dividing the shape into identical parts that are mirror images of one another.**
- **The number of axis of symmetry of a shape or figure depends on the number of times a shape or figure can be divided into two identical parts.**

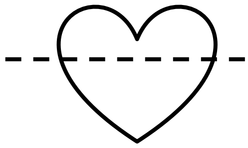
NOW DO PRACTICE EXERCISE 1

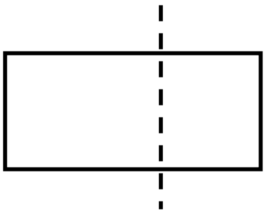
Practice Exercise 2

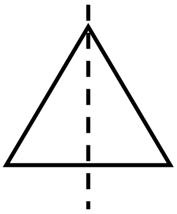
1. Is the line drawn an axis or line of symmetry for the figure below?

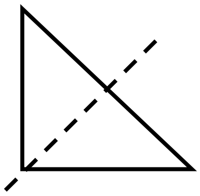
(a)


(b)


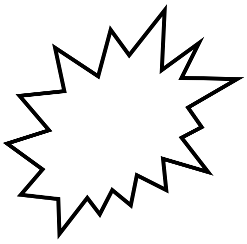
(c)


(d)


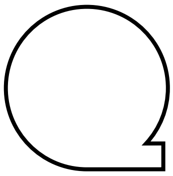
(e)


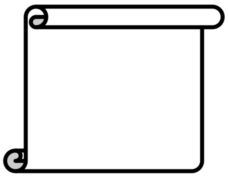
(f)


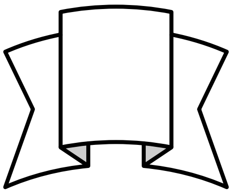
2. Are these figures symmetrical? If YES, draw an axis or line of symmetry.

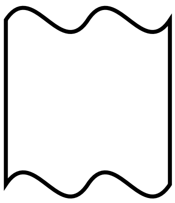
(a)


(b)


(c)


(d)


(e)


(f)


3. Does a parallelogram have an axis of symmetry?

4. Name the quadrilaterals which are symmetrical about:

(a) one diagonal only
(b) both diagonals
(c) neither diagonal

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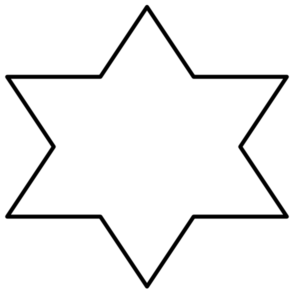
5. This figure has line symmetry about six (6) axes. Sketch the figure and draw in the axes of symmetry.



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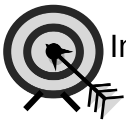
6. Write the capital letters of the alphabet which has line symmetry. Draw the axis or line of symmetry to them.

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 1.

Lesson 3: Rotational Symmetry

You have learnt to identify symmetry in figures and define line symmetry in Lesson 2.



In this lesson you will:

- describe rotational symmetry
- work out order of rotational symmetry of shapes and objects around us.

You learnt something about rotational symmetry in Grade 8 Mathematics. This section will give you further knowledge and skills regarding rotational symmetry.

If a figure matches itself a number of times while it is being turned about a point, then it is said to have **Rotational Symmetry.**

Example 1

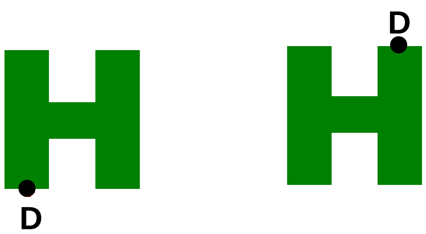
Look at these figures. They appear to be unchanged even after the figures are rotated. Hence they have rotational symmetry.

If you can rotate (or turn) a figure around a centre point by fewer than 360° and the figure appears unchanged, then the figure has **rotational symmetry**.

The point around which you rotate is called the **centre of rotation**, and the smallest angle you need to turn is called the **angle of rotation.**

Example 2

When the letter “H” below is rotated about the point D by 180º, it exactly fits on the original figure.

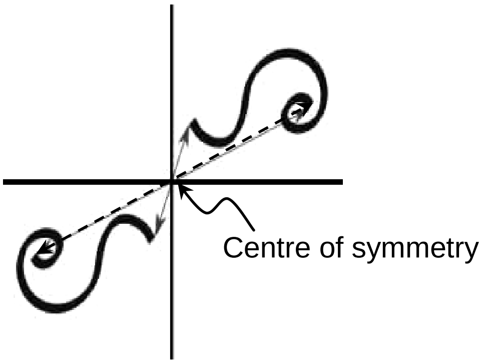


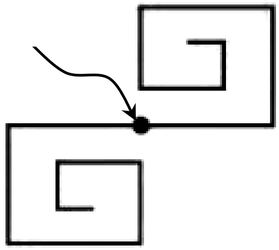
So the letter “H” has a rotational symmetry.

If a figure is constant under rotation through a half turn of 180º we have a special type of rotational symmetry which gives useful results. Such symmetry about a point is called **point symmetry** and the centre of rotation is called the **centre of symmetry**.

Note: Point Symmetry is sometimes called **Origin Symmetry,** because the "Origin" is the central point about which the shape is symmetrical.

See the figures below.



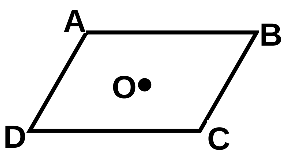


They look the same when viewed from opposite directions, such as left versus right, or if turned upside down.

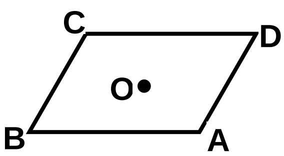
> **Point symmetry is when every part has a matching part. The same distance from the centre point but in the opposite direction.**
>
> **It is the same as Rotational symmetry of order 2.**

Example 3

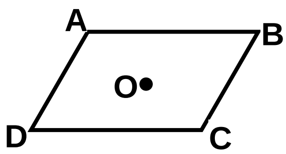
Here is Parallelogram ABCD.



Parallelogram ABCD can be rotated through 180º and 360ºabout the Point O.



Position when rotated through 180º



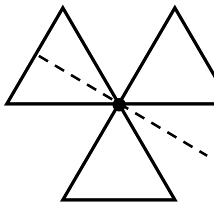
Position when rotated through 360º

There are only two positions in which the shape looks identical to how it is started.

You can say that Parallelogram ABCD has rotational symmetry **of order 2 about point** O.

If a shape only matches itself once as you go around (meaning it matches itself after one full rotation of 360º) there is really **no symmetry** at all.

For example this shape has no symmetry at all.



No matching point on other side of the centre the same distance away.

As we have learnt, a figure has rotational symmetry if we can rotate it around its centre and get the same figure.

For example

If you take a square and rotate it 90º, you still get the same square. We can calculate this 90º by just dividing 360º by the number of sides of the square which is 4. And we say the square has rotational symmetry every 90º. (90º, 180º and 270º)
So to find the angle of rotation, we use the formula:

$$\textbf{Angle of rotation} = \frac{\textbf{360}^\circ}{\textbf{number of sides}}$$

Below are examples of figures with their calculated angle of rotations.

1) A pentagon (5 sides) has rotational symmetry every $\frac{360^\circ}{5}$ = 72 degrees

2) A hexagon (6 sides) has rotational symmetry every $\frac{360^\circ}{6}$ = 60 degrees

3) A heptagon (7 sides) has rotational symmetry every $\frac{360^\circ}{7}$ = 51.4 degrees

4) An octagon (8 sides) has rotational symmetry every $\frac{360^\circ}{8}$ = 45 degrees

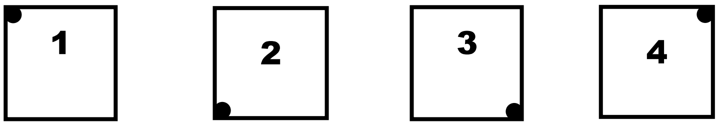
5) A triangle (3 sides) has rotational symmetry every $\frac{360^\circ}{3}$ = 120 degrees

The angle of rotation will give us the **order of rotational symmetry**.

The order of rotational symmetry is the number of positions a figure can be rotated to, without bringing any changes to the way it looks originally.

Example of Order of Rotational Symmetry

1. Here is a square.

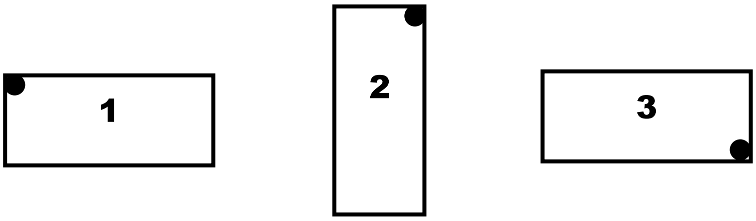


Notice that apart from the dot, the square looks exactly the same in 1, 2, 3, and 4.

We say that the square has got rotational symmetry of order 4.

This means that there are four (4) positions in which the square looks exactly the same upon rotation.

2. Here is a rectangle.

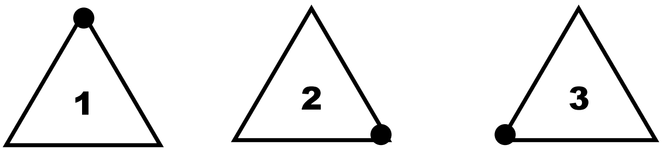


Notice that apart from the dot, the rectangle looks exactly the same in 1 and 3.

We say that the rectangle has got rotational symmetry of order 2.

This means that there are two (2) positions in which the rectangle looks exactly the same upon rotation.

3. Here is an equilateral triangle.



Notice that apart from the dot, the equilateral triangle looks exactly the same in 1, 2 and 3.

We say that the equilateral triangle has rotational symmetry of order 3.

This means that there are two (2) positions in which the equilateral triangle looks exactly the same upon rotation.

Order of rotational symmetry depends upon the number of equal sides that a regular polygon has.

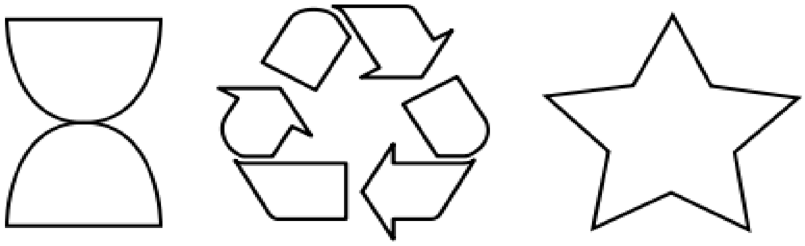
A figure has order **n** rotational symmetry if $\frac{1}{n}$ of a complete turn (360º) leaves the figure unchanged. Another way to say this is that the figure has **n**-fold rotational symmetry.

The order of rotation corresponds to a $\frac{360^\circ}{n}$ angle of rotation.

So to find the order of Rotational symmetry when the angle of rotation is given, we used the formula:

$$\textbf{Order of rotation symmetry} = \frac{\mathbf{360^\circ}}{\textbf{angle of rotation}}$$

Example Find the order of rotational symmetry of the figures below.



Solutions:

The figure on the left can be turned by 180° (the same way you would turn an hourglass) and will look the same.

So, the order of rotation = $\frac{360^\circ}{180^\circ} = \mathbf{2}$

The center (recycle) figure can be turned by 120°

So, the order of rotation = $\frac{360^\circ}{120^\circ} = \mathbf{3}$

The star has five points. To rotate it until it looks the same, you need to make $\frac{1}{5}$ of a complete 360° turn. Since, $\frac{1}{5}$ of 360º = 72º, this is a 72° angle rotation.

So, the order of rotation = $\frac{360^\circ}{72^\circ} = \mathbf{5}$

Here are examples of different rotational symmetry orders in real world.

Red knobbed starfish

Clematis flower

Spiderwort flower

1. Spiderwort flower for instance has three leaves arranged around the center of the flower. The Spiderwort has 3-fold rotational symmetry (120º)

2. The red knobbed starfish shown here has five equally spaced legs. It has 5-fold rotational symmetry (72º).

3. The Clematis shown has 8-fold rotational symmetry (45º). It has 8 flower petals arranged around the center of the flower.

You can check for instance that with the spiderwort flower, we have an order 3 rotational symmetry and the angle of rotation would be found by dividing 360º by 3.

Solution: Angle of rotation = $\frac{360^\circ}{3} = 120^\circ$

In the same way you can also check for the red knobbed starfish and the Clematis flower.

For the red knobbed starfish:

Solution: Angle of rotation = $\frac{360^\circ}{5} = 72^\circ$

For the Clematis flower:

Solution: Angle of rotation = $\frac{360^\circ}{8} = 45^\circ$

NOW DO PRACTICE EXERCISE 3

Practice Exercise 3

1. State the order of rotational symmetry of each shape below.

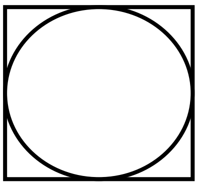
(a) H

(b) T

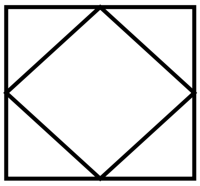
(c) P

(d) X

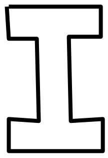
(e)



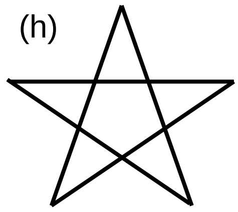
(f)



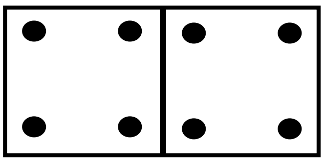
(g)



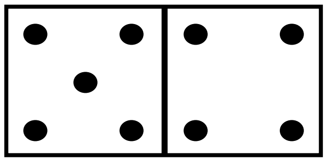
(h)



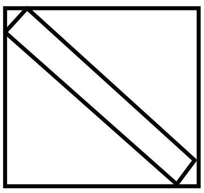
(i)



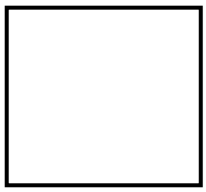
(j)



(k)



(l)

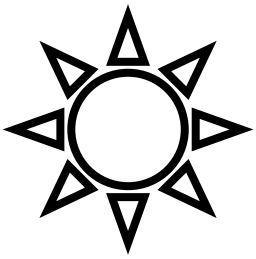


2. What do you think is the order of rotational symmetry of the capital letter drawn below? Explain why your answer.

A

3. Find the angle of rotation for the image below?

(a)



(b)

(c)

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4. Each of the figure below has rotational symmetry. Find the centre of rotation and the angle of rotation.

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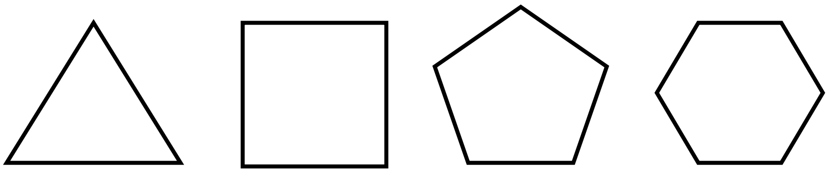
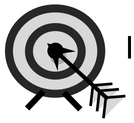
5. For each of the following polygons, what is the centre of rotation and angle of rotation?



CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 1

Lesson 4: Scale Drawing

You learnt to identify rotational symmetry in figures.in the last lesson. You also learnt to work out the order of rotational symmetry and angle of rotation of shapes and object around us.



In this lesson, you will:

- define scale drawing
- use scale factors or enlargement factors to increase and decrease sizes of objects.

There are situations where we have to use scale drawings in everyday life.

Almost everyone is familiar with toys that are scale models of cars, airplanes, boats and many other familiar objects. When someone plans a trip, it is convenient to have a map of the place where the trip will take place.

Scale drawings and models are also very important for architects, builders, carpenters, plumbers, engineers, electricians and fashion designers. Architects and designers works in the same manner. They start with sketches of ideas showing what they want their buildings to look like, then, translate to accurate scale drawings.

A scale drawing is a drawing that shows a real object with accurate sizes except they have all been reduced or enlarged by a certain amount (called the scale).

Example

A map cannot be of the same size as the area it represents. So, the measurements are **scaled down** to make the map of a size that can be conveniently used by users such as motorists, cyclists and bushwalkers.

A scale drawing of a building (or bridge) has the same shape as the real building (or bridge) that it represents but a different size. Builders use scaled drawings to make buildings and bridges.

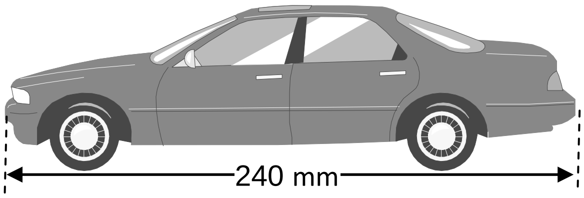
A ratio is used in scale drawings of maps and buildings. That is:

The scale of a Drawing = Drawing Length : Actual Length

The **scale** is shown as the length in the drawing, then a **colon** (**:**)**,** then the matching length on the real thing.

Example:

Since it is not always possible to draw on paper the actual size of real-life objects such as the real size of a car, an airplane, a butterfly, a horse, a house, we need scale drawings to represent the size like the one you see below of a car.



.
In real life, the length of the car above may measure 240 cm. However, the length of a copy or print paper that you could use to draw this car is a little bit less than 12 cm.

Since $\frac{240}{12}$ = 20, you will need about 20 sheets of copy paper to draw the length of the actual size of the car.

In order to use just one sheet, you could then use I cm on your drawing to represent 20 cm on the real-life object.

You can write this situation as : 1 **:** 20 or $\frac{1}{20}$ or 1 is to 20.

Notice that the first number always refers to the length of the drawing on paper and the second number refers to the length of real-life object.

Example1

Suppose a problem tells you that the length of a vehicle is drawn to scale. The scale of the drawing is 1**:**20.

If the length of the drawing of the vehicle on paper is 12 cm, how long is the vehicle in real life?

Set up a proportion that will look like this: $\frac{\text{Length of drawing}}{\text{Real Length}} = \frac{1}{20}$

Do a cross product by multiplying the numerator of one fraction by the denominator of the other fraction.

We get: Length of drawing × 20 = Real Length × 1

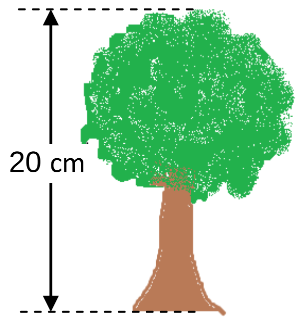
Since, length of drawing = 12, we get 12 x 20 = Real Length x 1

240 = Real Length

Therefore, Real Length is 240 cm.

Example 2

If the height of the tree on paper is 20 cm, what is the height of the tree in real life if the scale of the drawing is 1: 500?



Solution:

Set up a proportion like this: $\frac{\text{Length of drawing}}{\text{Real Length}} = \frac{1}{500}$

Do a cross product by multiplying the numerator of one fraction by the denominator of the other fraction.

We get: Length of drawing × 500 = Real Length × 1

Since, length of drawing = 20, we get 20 x 500 = Real Length x 1

1000 = Real Length

Therefore, Real Length is 1000 cm.

Example 3

The scale of the picture is 12 **:** 3 cm. Find the actual length of the flask.

Solution:

Let **n** be the actual length of the flask.

12**:**3 = 4: n	[Write the proportion.]
n × 12 = 4 × 3	[Write the cross products.]
n = 24	[Simplify.]

The actual length of the flask is 24 cm.

The ratio of the length of the scale drawing to the corresponding length of the actual object is called as Scale Factor.

In formula;

$$\textbf{Scale factor} = \frac{\text{Length of drawing}}{\text{Length of actual object}}$$

Uses of the scale factor

- A scale factor is a number used as a multiplier in scaling.
- A scale factor is used to scale shapes in 1, 2, or 3 dimensions.

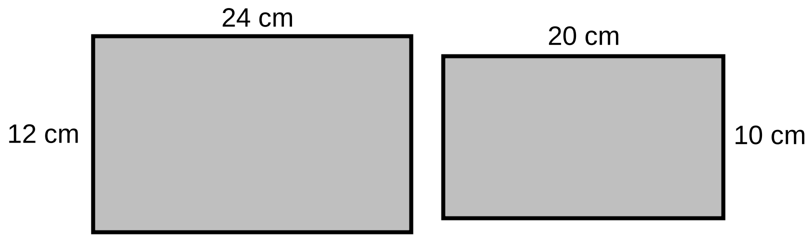
The scale factor can be seen in the following

1. **Size Transformation:** In size transformation, the scale factor is the ratio of expressing the amount of magnification.
2. **Scale Drawing:** In scale drawing, the scale factor is the ratio of measurement of the drawing compared to the measurement of the original figure.
3. **Comparing Two Similar Geometric Figures:** The scale factor when comparing two similar geometric figures is the ratio of lengths of the corresponding sides.

Problem Solving on Scale Factor

Example 1

Find the scale factor from the larger rectangle to the smaller rectangle, if the two rectangles are similar.



Solution:

Step 1: If we multiply the length of one side of the larger rectangle by the scale factor we get the length of the corresponding side of the smaller rectangle.

Step 2: Dimension of larger rectangle × scale factor = dimension of smaller rectangle

Step 3: 24 × scale factor = 20 [Substitute the values.]

Step 4: Scale factor = $\frac{20}{24}$ [Divide each side by 24.]

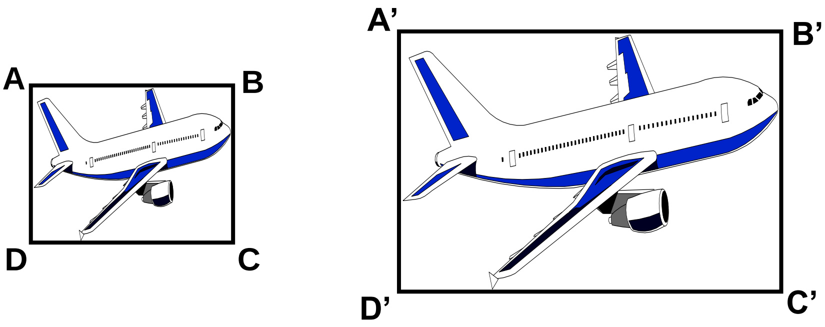
Step 5: Scale factor = $\frac{5}{6}$ = 5:6 [Simplify.]

Therefore, scale factor from the larger rectangle to the smaller rectangle is 5:6.

Example 2

On the figure AB = 2 cm. On the image A'B' = 5 cm.

What is the scale factor of this enlargement?



Solution:

$$\text{Scale Factor} = \frac{\text{Length of drawing}}{\text{Legnth of actual object}}$$

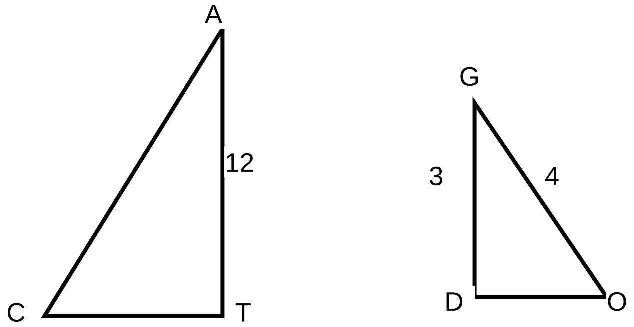
$$\text{Scale factor} = \frac{A'B'}{AB} = \frac{5\text{ cm}}{2\text{ cm}}$$

$$= 2.5$$

NOW DO PRACTICE EXERCISE 4

Practice Exercise 4

1. Use the diagrams below to answer Questions (a) below.

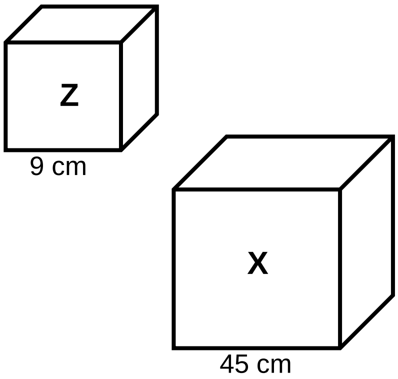


(a) What is the scale factor from triangle CAT to triangle DOG?

2. A model of a dinosaur skeleton was made using a scale of 1 cm **:** 15 cm in a museum. If the size of the dinosaur's tail in the model is 8 cm, then find the actual length of dinosaur's tail.

3. The scale of the picture shown is 1 cm **:** 3 inches. Find the actual height of the cup.

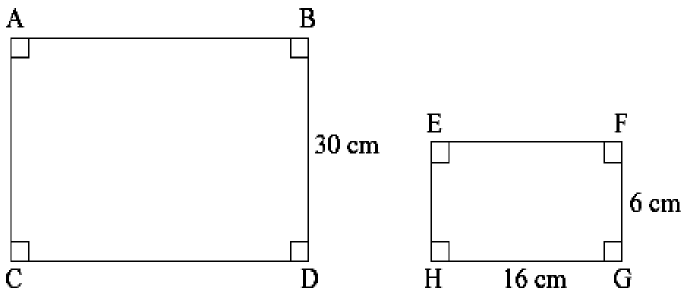
4. On the diagram Cube X is an enlargement of Cube Z.



What is the Scale factor of this enlargement?

---

5. The following diagram shows two similar rectangles:

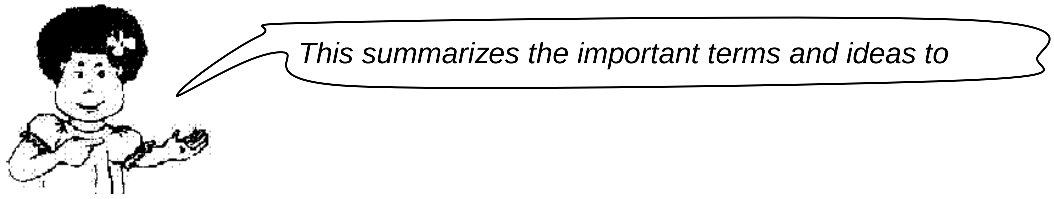


(a) What is the scale factor from rectangle EFGH to rectangle ABCD?

(b) What is the length of side CD?

CORRECT YOUR WORK ANSWERS ARE AT THE END OF TOPIC 1

TOPIC 1: SUMMARY



- The word "**transformation**" refers to a movement or change in a geometric shape. There are four main types of transformation:
  a) **Sliding transformation (Translation)** – the object is moved by sliding. Every point moved in the same distance and in the same direction. We can write this translation using the **vector notation:** $\begin{pmatrix} \pm \mathbf{x} \\ \pm \mathbf{y} \end{pmatrix}$
  b) **Spin Transformation (Rotation)** – the original object is moved by spinning or turning it about a point. Every point of the object or shape turns through the same angle about a point.
  c) **Turn over Transformation (Reflection)** – The original object is changed into its mirror image. The object and its image are oppositely congruent.
  d) **Size Transformation (Enlargement or Reduction)** – the original object is changed in size but its shape stays the same. To carry out an enlargement requires two pieces of information:
     (i) The **Scale factor**-the number used a a multiplier in size transformation.
     (ii) The **centre of enlargement** – the point around which the object is enlarged.
- **Symmetrical Shapes** are shapes or objects that have two halves that are congruent (exactly the same size and shape).
- **The Line or Axis of Symmetry** is the line dividing the shape into two identical parts that are mirror images of one another.
- **The Rotational Symmetry** is symmetry wherein a figure matches itself a number of times while it is being turned about a point. The point around which you rotate is called **centre of rotation** and the smallest angle you need to turn is called **angle of rotation**.
- **The order of rotational symmetry** is the number of positions a figure can be rotated, without bringing any changes to the way it looks originally in a full turn.
- **A scale drawing** is a drawing that shows a real object with accurate sizes except they have all been reduced or enlarged by a certain amount (called the scale).
- **The Scale Factor** is the ratio of the length of the scale drawing to the corresponding length of the actual object.

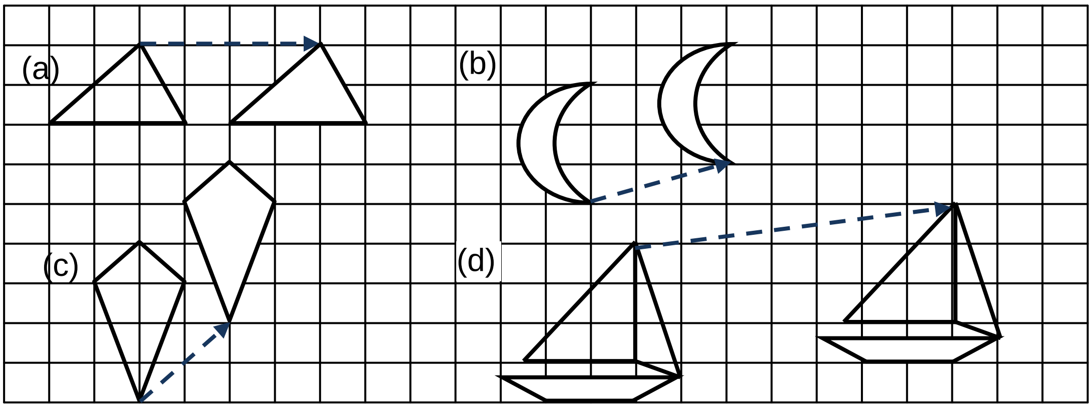
In formula; **Scale factor =** $\dfrac{\textbf{Length of drawing}}{\textbf{Legnth of actual object}}$

REVISE LESSONS 1- 4. THEN DO TOPIC TEST 1 IN ASSIGNMENT 6

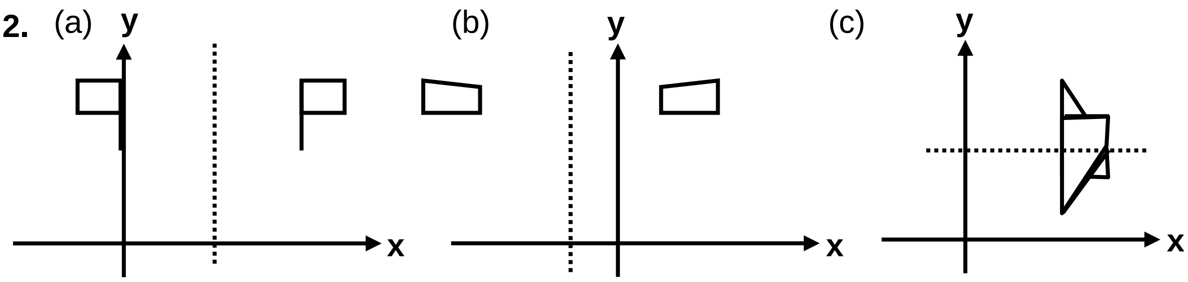
ANSWERS TO PRACTICE EXERCISES 1- 4

Practice Exercise 1

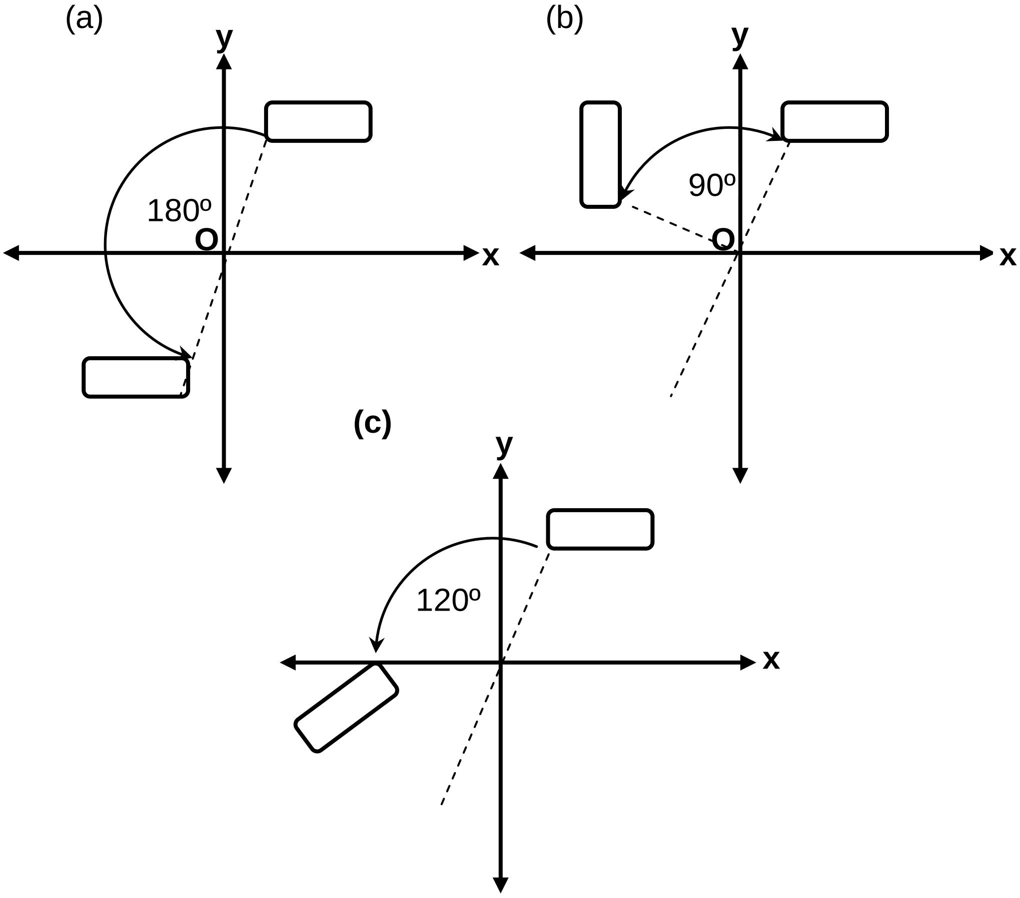
1.



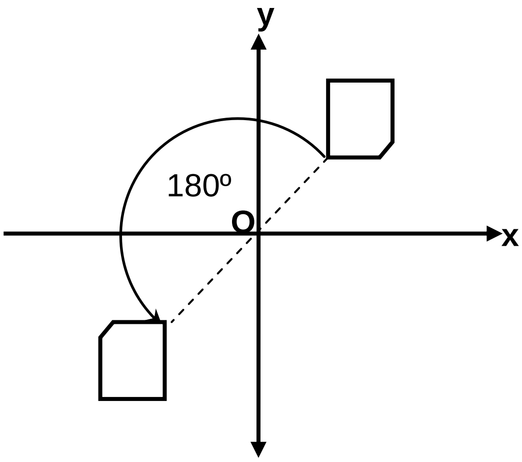
2.



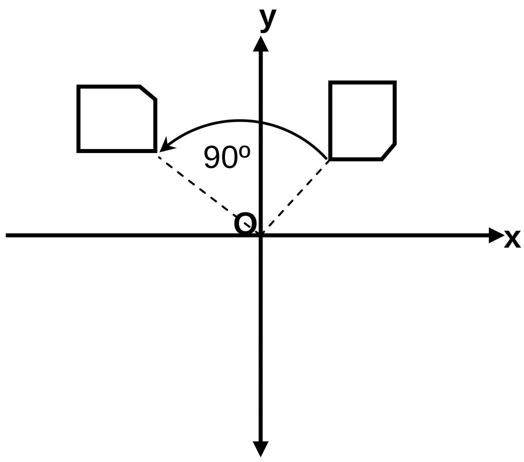
3. (i)



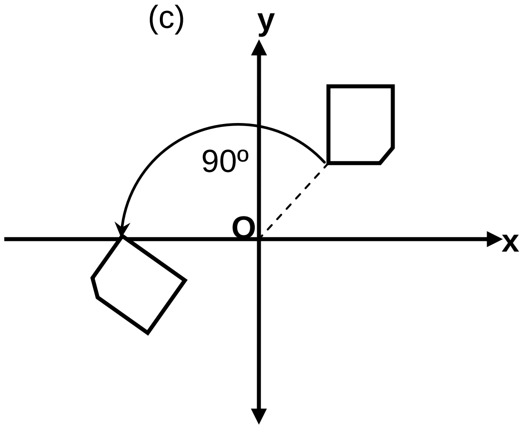
(ii) (a)



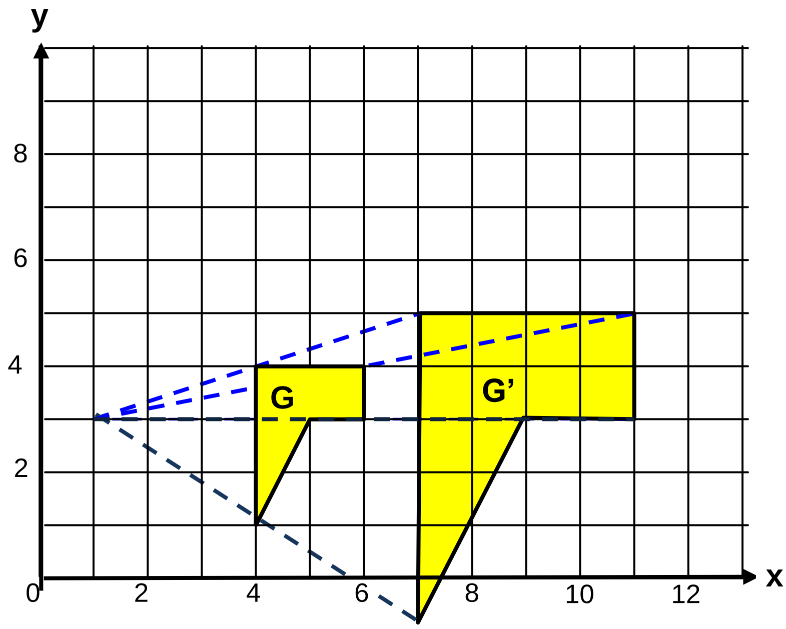
(b)



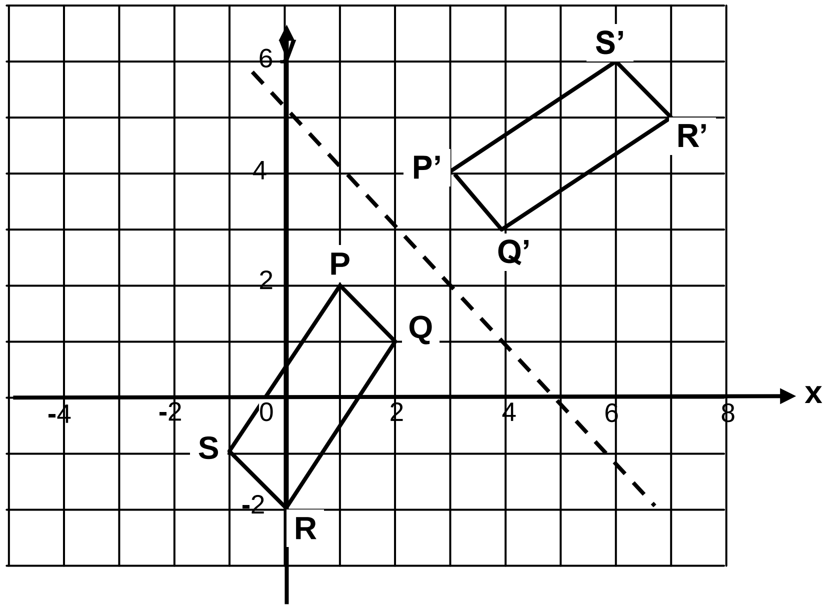
(c)



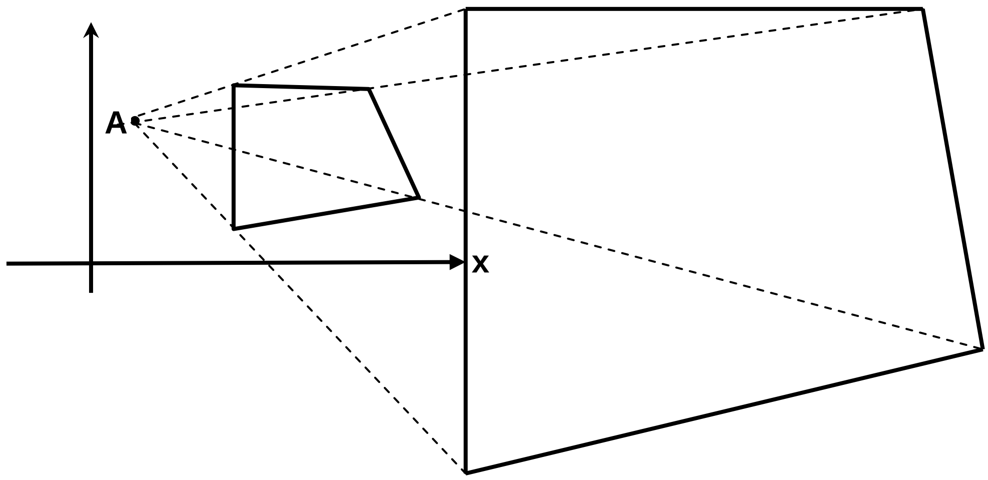
4.



5. .

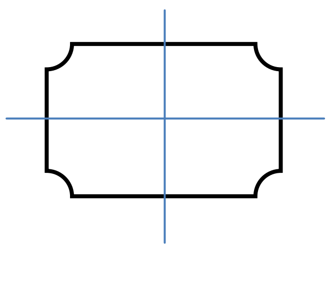


6.

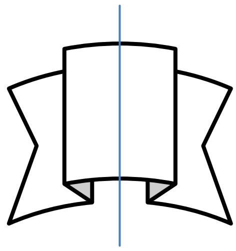


Practice Exercise 2

1.	(a)	No	(b)	Yes	(c)	No
	(d)	No	(e)	Yes	(f)	Yes
2.	(a)	No	(b)	Yes	(c)	No



(d) No (e) Yes (f) No



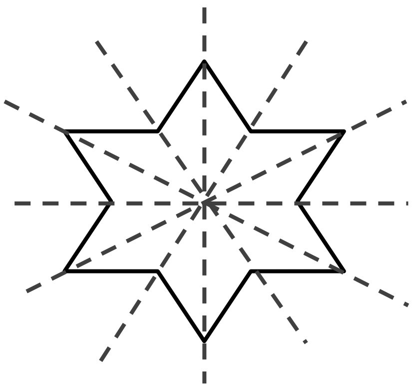
3. No. it has no line of symmetry because the side do not match when folded.

4. (a) kite,

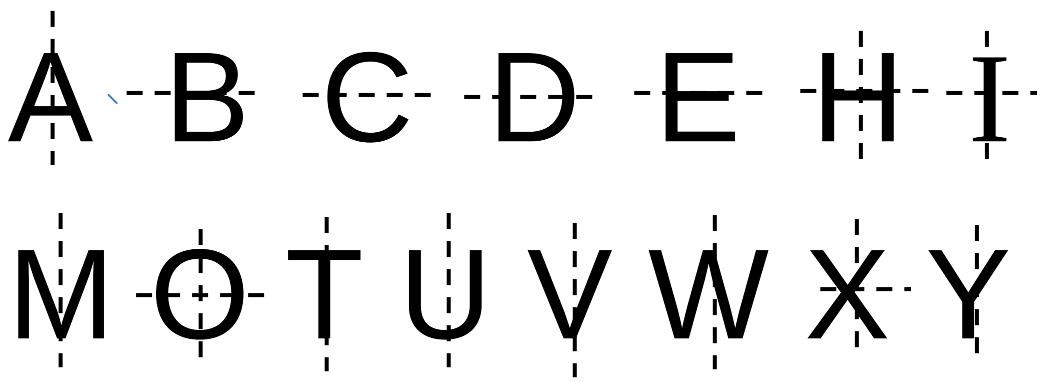
(b) Square; rhombus

(c) Rectangle, parallelogram, trapezium

5.



6.



Practice Exercise 3

1. (a) 2 (b) 2 (c) 1 (d) 2 (e) 4 (f) 4

(g) 2 (h) 5 (i) 2 (j) 1 (k) 2 (l) 4

2. No symmetry. Because it only matches itself once as you go around (meaning it matches itself after one full rotation of 360º) there is really **no symmetry** at all.

3. (a) 45º (b) 90º (c) 180º

4. For each figure the angle of rotation is the centre.

   The angles of rotations are: 120º, 180º, 120º, 90º

5. For each figure the angle of rotation is the centre of the polygon

   The angles of rotations are: 120º, 90º, 72º, 60º

Practice Exercise 4

1. $\frac{1}{4}$

2. 120 cm

3. 6 inches

4. 5

5. (a) 5 (b) 80 cm

END OF TOPIC 1

TOPIC 2

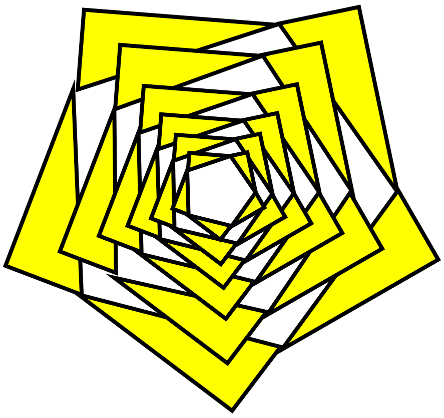
SIMILARITY AND CONGRUENCE

TOPIC 2: SIMILARITY AND CONGRUENCE

Introduction

Recognizing and using congruent and similar shapes can make calculations and design work easier.

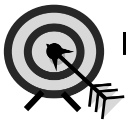
For instance, in the design below, only two different shapes were actually drawn. The design was put together by copying and manipulating these shapes to produce versions of them of different sizes and in different positions.



In this Topic, we will look in a little more depth at the mathematical meaning of the terms similar and congruent, which describes the relation between shapes like those in this design.

Lesson 5: Similar Figures

You have learnt something about Similar Shapes in your Gr 8 Mathematics. Also in the last lesson of this unit, you learnt about scale drawing, which is also a skill used in producing similar figures.



In this lesson you will:

- define similar figures
- identify similar figures and use the notation to denote them.
- identify corresponding angles of similar figures.

Similar figures are figures that are the same but of different sizes. Similar figures have their sides all increased or decreased in the **same ratio**, but their angles remain the same. These matching angles formed are equal and they are also referred to as **corresponding angles**. We will look at this in more details later in this lesson.

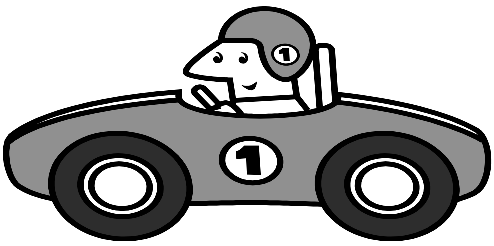
Properties of similar figures:

1. The corresponding angles of similar figures are equal.
2. The lengths of the corresponding sides of similar figures are in the same ratio.
3. The value of this ratio is equal to the scale factor, *k*.

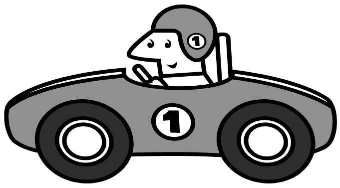
When a shape is enlarged or reduced, a similar figure is formed. In Lesson 4 of this unit, you have learnt about scale drawing.

A Scale Drawing is a drawing that shows a real object with accurate sizes except they have all been reduced or enlarged by a certain amount (called the scale).

Scale drawing is a skill used to produce scale diagrams which are examples of similar figures, as are scale model of cars. They have the same shape but are different in size.



Model 2

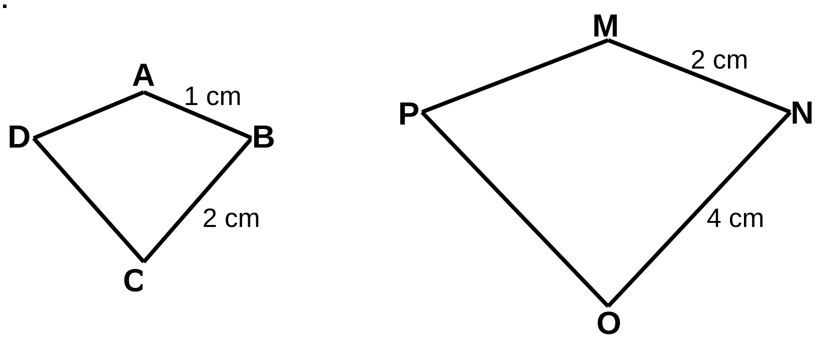


Model 1

The car 'Model 2' is similar to car 'Model 1'. It has a shape that is the same as that of 'Model 1'. But the car in Model 2 is bigger than the car Model 1 in size.

Let’s have a look at two kites.

Two kites are drawn, one with the top two side lengths 1 cm and bottom two side lengths 2 cm and the other with the top two side lengths 2 cm and bottom two side lengths 4 cm.



These figures are similar because they are of the same shape and have all corresponding angles equal and all corresponding sides in the same proportion. In the same proportion meaning that their sides are increased or decreased in the same ratio.

Corresponding Angles

As mentioned earlier in this lesson, corresponding angles are equal. Hence, in the above diagram, the pairs of corresponding angles are:

$\angle$**ABC** is correspondent to $\angle$**MNO**, which means that $\angle$**ABC** = $\angle$**MNO**

$\angle$**BCD** is correspondent to $\angle$**NOP**, which means that $\angle$**BCD** = $\angle$**NOP**

$\angle$**ADC** is correspondent to $\angle$**MPO**, which means that $\angle$**ADC** = $\angle$**MPO**

$\angle$**DAB** is correspondent to $\angle$**PMN**, which means that $\angle$**DAB** = $\angle$**PMN**

Corresponding angles are equal.

Therefore, Kite ***ABCD*** ||| Kite ***MNOP***

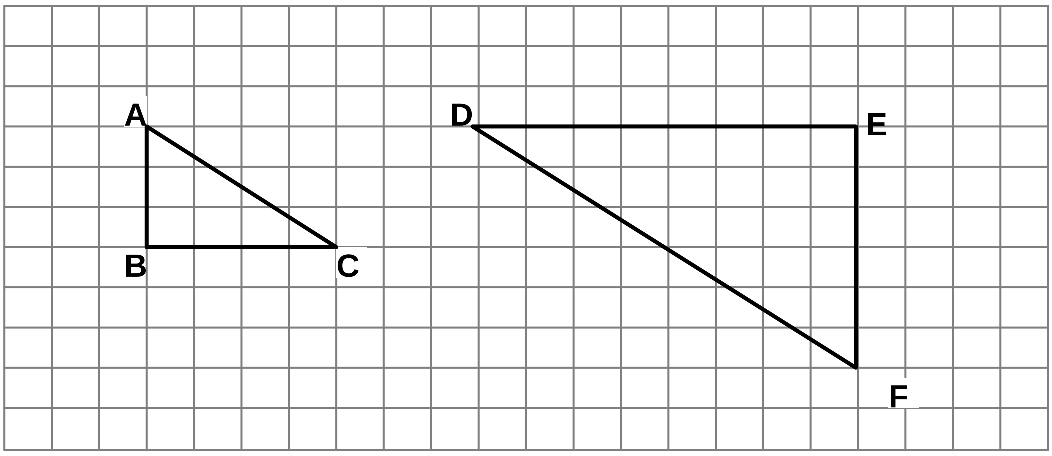
The symbol for ‘is similar to’ is |||. In most text the symbol, $\sim$, is used instead. Either of these notations is used to denote similarity.

Notation is another word for symbol. The notation used to denote similarity is $\sim$.

A grid can also be used to draw similar figures.

Here are examples of similar figures drawn using grid.

1) These two triangles are similar.



(a) Measure each angle in i. $\triangle ABC$ ii. $\triangle DEF$

In the above diagram,

$\angle$**A** is correspondent to $\angle$**F,** $\angle$***A*** = $\angle$**F** = 53º

$\angle$**B** is correspondent to $\angle$**E**, $\angle$**B** = $\angle$**E** = 90º

$\angle$**C** is correspondent to $\angle$**D,** $\angle$**C** = $\angle$**D** = 37º

(b) Measure of side lengths.

$\triangle$**ABC**	$\triangle$**DEF**	**Ratio of the sides**
Side AB = 3 cm	Side EF = 6 cm	$\frac{EF}{AB} = \frac{6}{3}$ = 2 cm
Side AC = 5 cm	Side DF = 10 cm	$\frac{DF}{AC} = \frac{10}{5}$ = 2 cm
Side BC = 4 cm	Side ED = 8 cm	$\frac{ED}{BC} = \frac{8}{4}$ = 2 cm

(c) It can be seen from the table above that the corresponding sides are in proportion. The ratio of sides gives the **enlargement factor**. In this case, the enlargement factor is 2.

(d) Now using the corresponding angles to complete the similarity statement;
$\triangle$ABC ||| $\triangle$EFD or $\triangle$ABC $\sim$ $\triangle$EFD.

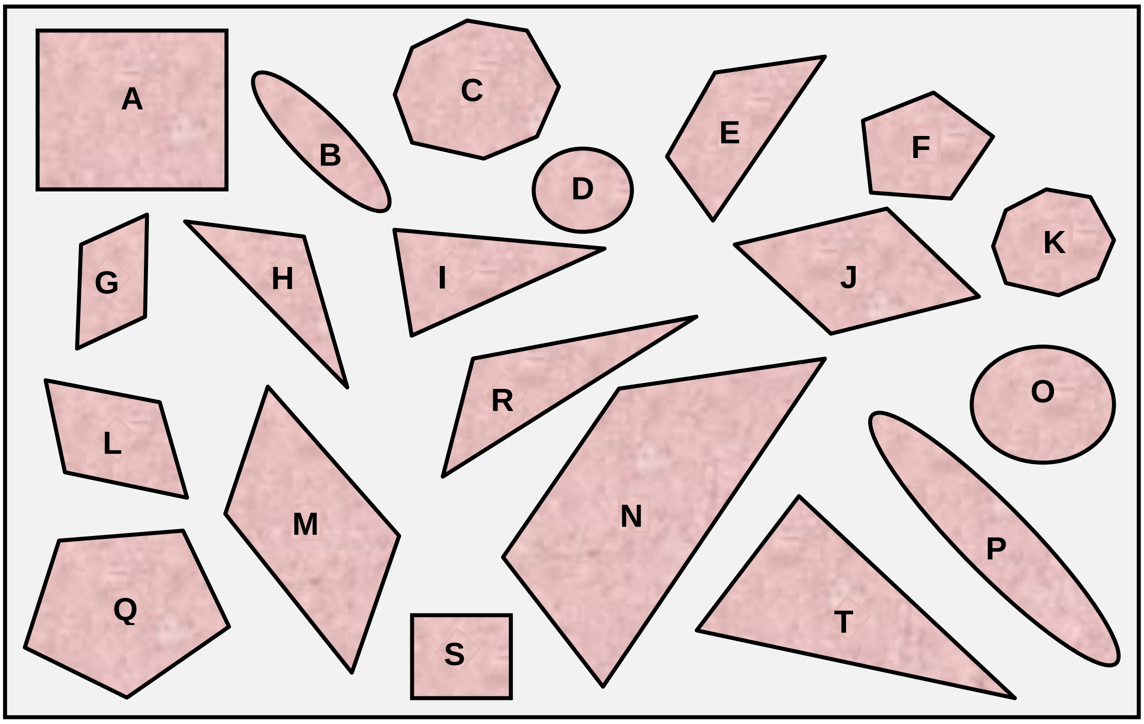
Enlargement factor is the ratio by which the corresponding sides are increased.

Reducing factor is the ratio by which the corresponding sides are decreased.

NOW DO PRACTICE EXERCISES 5

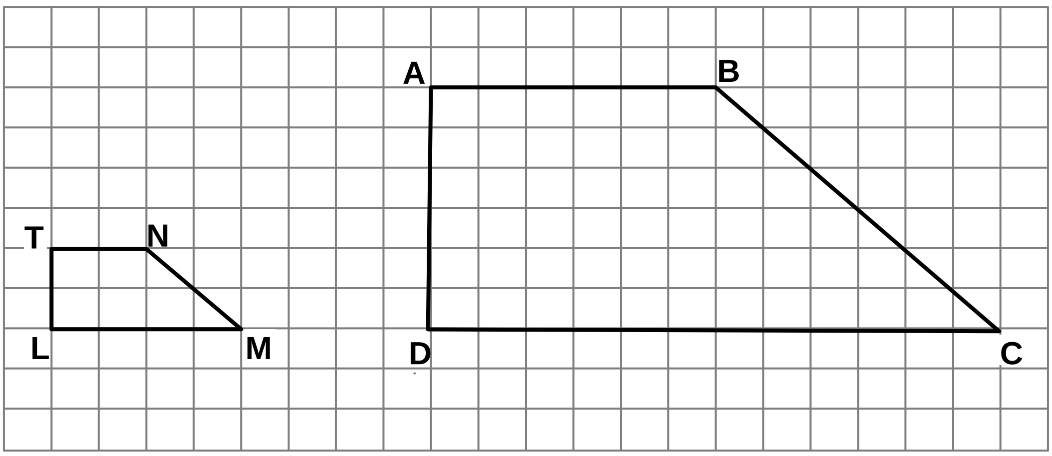
Practice Exercise 5

1. Select the pairs of similar figures. They have the same shape but different sizes.



Answers:

2. Refer to these two similar trapeziums to answer the questions **a** to **f** on the next page.



(a) Measure each angle in i. Trapezium *TNML* ii. Trapezium *ABCD.*

(i) $\angle$ **T** = _______ $\angle$ **A** =________

(ii) $\angle$ **N** = _______ $\angle$ **B** =________

(iii) $\angle$ **M** =_______ $\angle$ **C** =________

(iv) $\angle$ **L** =_______ $\angle$ **D** =________

(b) Which angles are corresponding?

(i) ___________=___________

(ii) ___________=___________

(ii) ___________=___________

(iv) ___________=___________

(c) Measure each of the sides. Copy and complete this table.

Trapezium ABCD	Trapezium TNML	Ratio of the sides
$\overline{AB}$ =_______________	$\overline{TN}$ =______________	$\frac{\overline{AB}}{\overline{TN}}$ =
$\overline{BC}$ =_______________	$\overline{NM}$ =______________	
$\overline{CD}$ =_______________	$\overline{ML}$ =______________	
$\overline{AD}$ =_______________	$\overline{TL}$ =______________	

(d) What do you notice about the corresponding angles?

Answer: ____

____

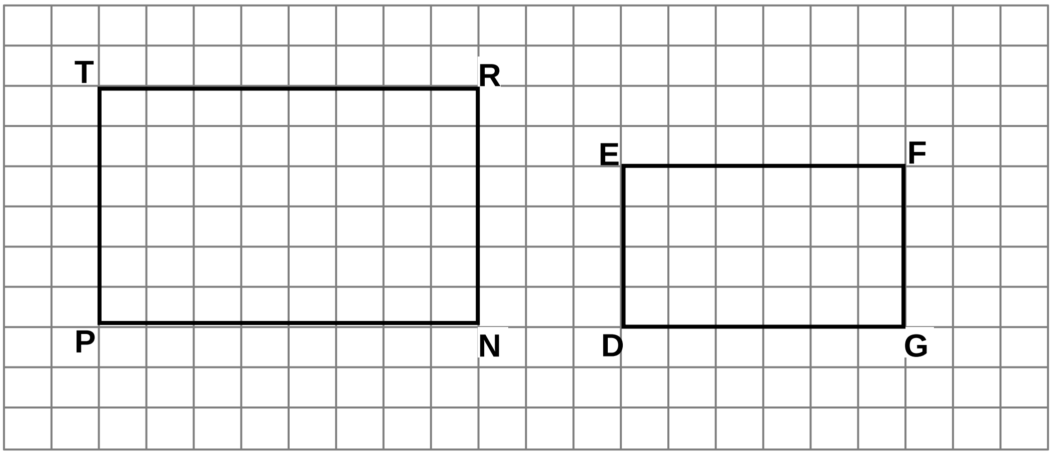
(e) What is the enlargement factor?

Answer:______________________________________

(f) Complete the similarity statement:

Trapezium **TNML** $\sim$ Trapezium __________

3. Two rectangles are drawn below.



(a) What can be said about the angles?

Answer: ____

____

(b) Measure each side and complete the table.

Rectangle **EFGD**	Rectangle **TRNP**	**Ratio of the sides**
$\overline{EF}$ = ______________	$\overline{TR}$ = ______________	$\frac{\overline{EF}}{\overline{TR}}$ =
$\overline{FG}$ = ______________	$\overline{RN}$ = ______________	
$\overline{GD}$ = ______________	$\overline{NP}$ = ______________	
$\overline{ED}$ = ______________	$\overline{PT}$ = ______________	

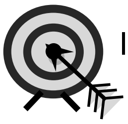
(c) These figures have corresponding angles equal but are not similar. Use part ‘b’ to give a reason why they are not similar.

Answer: ____

CHECK YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 2.

Lesson 6: Similar Triangles

In the last lesson, you learnt about similar figures. You will still be looking at similarity in this lesson. However, this lesson will focus only on the similarity of one particular shape, the triangles.



In this lesson, you will:

- define similar triangles
- identify similar triangles and use the notation to denote similar triangles.
- identify corresponding angles and corresponding sides of similar triangles and label them correctly.
- Solve problems involving similar triangles.

In your Grade 7 and Grade 8 mathematics, you learnt about the three different types of triangles and their properties. This lesson is on triangles as well, but it will mainly be focussing on the similarity of triangles.

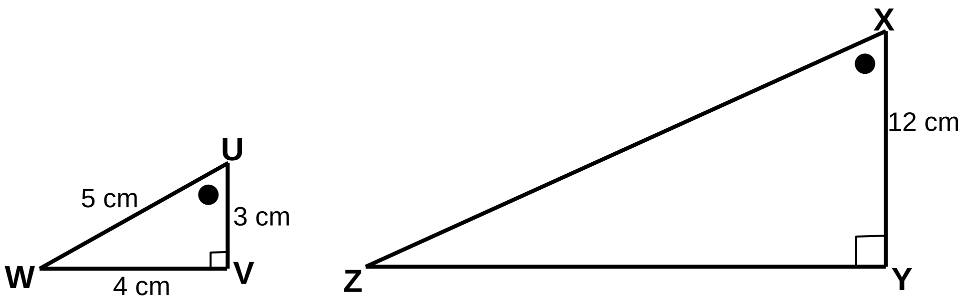
So, what are similar triangles?

Two triangles are said to be **similar** if the corresponding angles are equal and the corresponding sides are in proportion. In other words, **similar triangles** are triangles that are the same but different sizes.

> **In general, if two triangles are similar, then the corresponding sides are in the same ratio.**

Example 1:

Look at these two triangles, Δ**UVW** and Δ**XYZ**. These two triangles are similar.



Solution:

From the diagram above, the following corresponding sides and corresponding angles can be drawn.

i. Corresponding sides: In similar triangles, the sides facing the equal angles are always in the same ratio.

- $\overline{\mathbf{UV}}$ corresponds to $\overline{\mathbf{XY}}$,
- $\overline{\mathbf{VW}}$ corresponds to $\overline{\mathbf{YZ}}$,
- $\overline{\mathbf{UW}}$ corresponds to $\overline{\mathbf{XZ}}$.

ii. Corresponding angles.

- $\angle\mathbf{U} = \angle\mathbf{X}$
- $\angle\mathbf{V} = \angle\mathbf{Y}$
- $\angle\mathbf{W} = \angle\mathbf{Z}$

Note that the **corresponding angles** are marked in the **same way.**

iii. Calculating the lengths of corresponding sides.

Write out the proportion or the ratio of sides. The ratio of sides gives the scale factor. Use the two given corresponding sides to determine the scale factor.

$$\frac{\overline{\mathbf{XY}}}{\overline{\mathbf{UV}}} = \frac{12}{3} = 4, \text{ and 4 is the scale factor.}$$

Now that the scale factor is determined, it can be used to find the two unknown sides of Δ **XYZ**. To find these two unknown sides, **XZ** and **YZ**, multiply the scale factor by their respective corresponding angles.

That is;

$$\begin{aligned}\overline{\mathbf{XY}} &= \overline{\mathbf{UV}} \text{ x scale factor}\\ &= 5 \text{ cm x } 4\\ &= 20 \text{ cm}\end{aligned}$$

Therefore, $\overline{\mathbf{XY}}$ = 20 cm

$$\begin{aligned}\overline{\mathbf{YZ}} &= \overline{\mathbf{VW}} \text{ x scale factor}\\ &= 4 \text{ cm x } 4\\ &= 16 \text{ cm}\end{aligned}$$

Therefore, $\overline{\mathbf{YZ}}$ = 16 cm

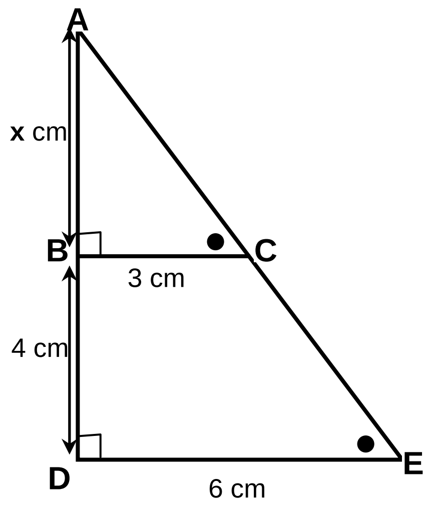
iv. Using the ‘similar to’ symbol, a similarity statement can be written for the two triangles.

Similarity statement: $\Delta\mathbf{UVW} \sim \Delta\mathbf{XYZ}$

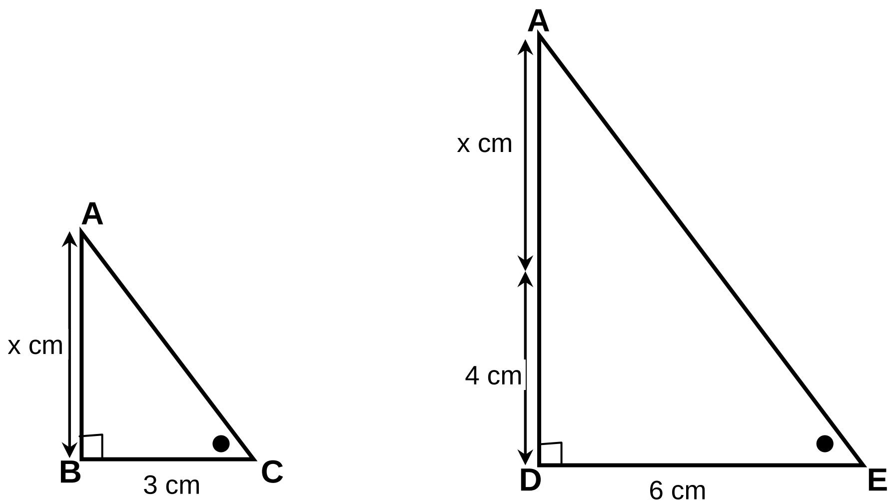
The symbol ($\sim$) is use to denote similarity.

Example 2

Study the diagram below. Can you see two similar triangles, ABC and ADE, in the diagram? Find the value of **x**, in the diagram.



Let's separate the two so that it gives a clear picture.



Triangle **ABC** Triangle **ADE**

Solution:

To solve for **x,** you must first of all identify the corresponding sides and get the proportion of the sides.

i. Corresponding sides:

$\overline{AB}$ corresponds to $\overline{AD}$,
$\overline{BC}$ corresponds to $\overline{DE}$ and
$\overline{AC}$ corresponds to $\overline{AE}$

ii. Proportion of sides: Determine the scale factor.

a. $\dfrac{\overline{DE}}{\overline{BC}} = \dfrac{6\text{ cm}}{3} = 2$, 2 is the scale factor.

b. Use the scale factor to solve for **x** in order to equate the ratio of the corresponding sides containing **x** to the scale factor, 2, and then solve. That is;

i. $\dfrac{\overline{\mathbf{AD}}}{\overline{\mathbf{AB}}} = 2$

$$\frac{(4+x)}{x} = 2$$

$x \times \left[\dfrac{(4+x)}{x}\right] = 2 \times x$, multiply x on both sides,

$$4 + x = 2x$$

$4 + x - x = 2x - x$, subtract x on both sides,

$4 = x$

or $x = 4$

Therefore, x = 4 cm.

ii. Substitute for the value of **x** to find the measure of sides **AB** and **AD.**

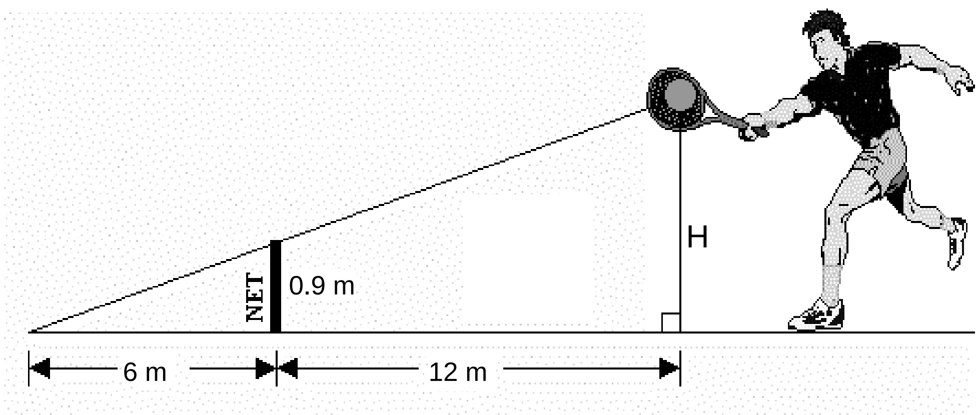
$\overline{\mathbf{AB}} = x = 4$ cm

$\overline{\mathbf{AD}} = 4 + x = 4 + 4 = 8$ cm

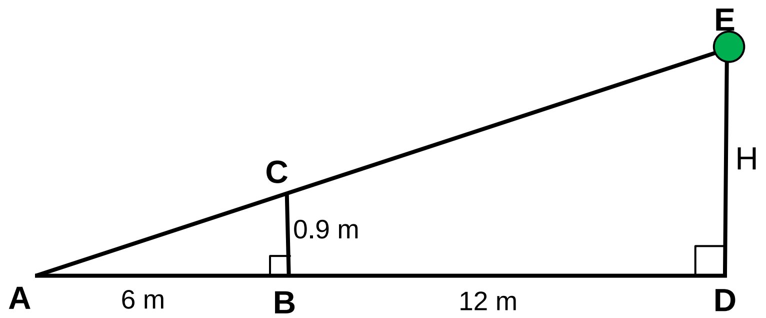
Similar triangles can be applied to solve real world problems.

For instance, consider a tennis player. The tennis player is 12 meters from the net and he must hit the ball at a certain height above the ground so that it will just pass over the net and land 6 meters away from the base of the net. At what height, **H**, in meters above the ground must he hit the ball?

Solution: When you have such practical problems, draw a diagram to help you solve the problem. For example, the problem above is illustrated in the diagram below.



i. Redraw and label the diagram as shown below. Height is measured vertically, so $\angle$**EDA** is a right angle, assuming that the net is vertical.



ii. Since the two triangles **ABC** and **ADE** are similar, find the scale factor and equate it with the ratio of sides $\overline{\mathbf{BC}}$ and $\overline{\mathbf{DE}}$ to solve for the height. That is;

$\overline{\mathbf{AD}} = \overline{\mathbf{AB}} + \overline{\mathbf{BD}} = 6 + 12 = 18$ cm.

(a) $\dfrac{\overline{\mathbf{AD}}}{\overline{\mathbf{AB}}} = \dfrac{18\text{ m}}{6\text{ m}} = 3$; and 3 is the scale factor.

(b) Now that the scale factor is determined, find H.

$$\frac{\overline{\mathbf{ED}}}{\overline{\mathbf{CB}}} = 3$$

$$\frac{H}{0.9\text{ m}} = 3$$ multiply 0.9 on both sides.

$$H = 3 \times 0.9$$

$$H = 2.7\text{ m}$$

So, the height (H) at which the ball must be hit is 2.7 m

Note:

Two triangles are similar if:

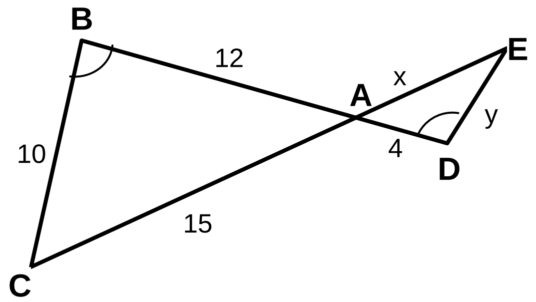
- **Two pairs of corresponding sides are in the same ratio and the angle included between them is equal.**
- **The corresponding sides are in the same ratio**
- **The corresponding angles are the same and equal.**

NOW DO PRACTICE EXERCISES 6

Practice Exercise 6

1. Identify the similar triangles in each figure and list the corresponding sides. Explain why they are similar and use the given information to find **x** and **y**.

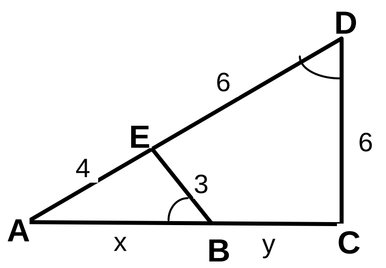
(a)



Similar triangles	Explanation	Corresponding sides

i. x =__________ y =__________

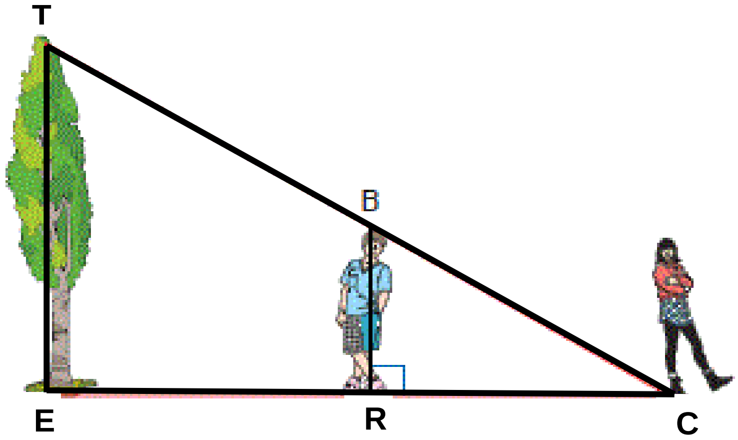
(b)



Similar triangles	Explanation	Corresponding sides

i. x =__________ y =__________

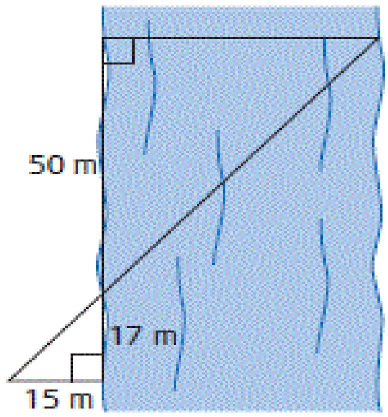
2. To measure the height of a tree, Cynthia has her little brother, **BR**, stands so that the tip of his shadow coincides with the tip of the tree's shadow, at point **C**.



Cynthia's brother, who is 1.2 m tall, is 4.2 m from Cynthia, who is standing at **C**, and 6.5 m from the base of the tree. Find the height of the tree..

---

3. Use the dimensions of the surveyors' triangle to find the width of the river, to the nearest meter.



(a) Redraw the diagram and label it.

(b) Identify the corresponding sides and use the scale factor to find the width of the river.

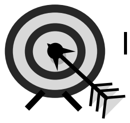
Calculation:

Therefore, the width of the river is:______________________

CHECK YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 2.

Lesson 7: Area of Similar Triangles

In the previous lesson, you learnt about similar triangles.



In this lesson, you will:

- identify steps in finding the area of similar triangles.
- calculate the area of similar triangles.
- solve problems involving areas of similar triangles.

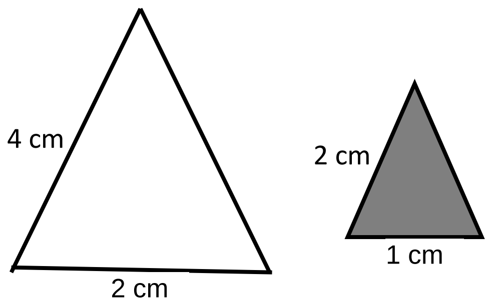
As discussed in the previous lesson, similar triangles have a common ratio for corresponding sides known as the scale factor, **k**. This scale factor, **k**, will be used in this lesson to determine the ratio of the areas of similar triangles which will be used to calculate the area of similar triangles.

If two similar triangles have corresponding sides in the ratio **x:y**, then their areas are in the ratio $\mathbf{x^2:y^2}$. This means that the ratio of the areas of two similar triangles is equal to the square of the ratio of corresponding sides. In other words, ratio of areas of two similar triangles is equal to the scale factor squared, $\mathbf{k^2}$.

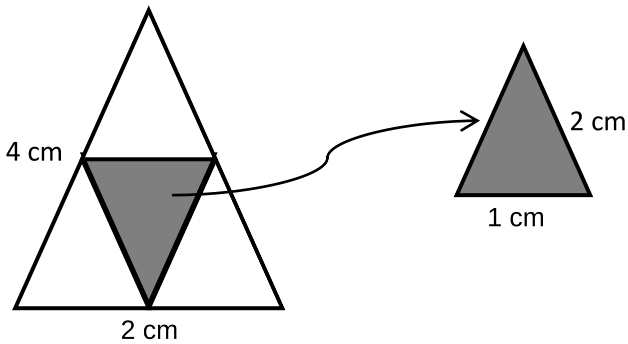
Ratio of Area of similar triangles

Example 1:

These two triangles are similar with sides in the ratio 2:1. This means that one of the triangles is twice as long as other. The scale factor, k = 2.



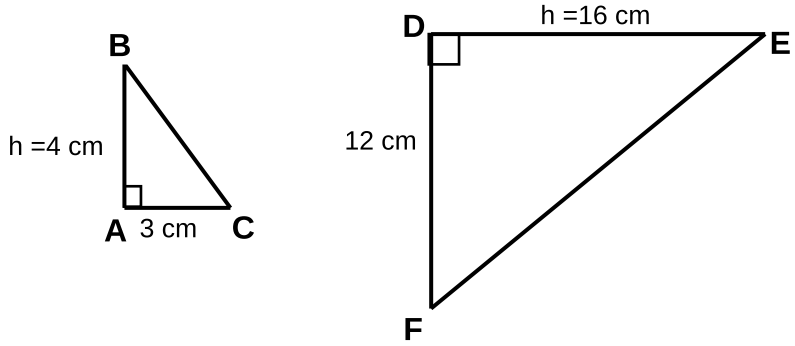
What can we say about their areas? The answer is simple if three more lines are drawn in.



You can see that the small triangle fits into the big triangle four times. So when the lengths are twice as long, their area is four times as big. The ratio of the area of the big triangle to the small triangle is 4:1. That is, $2^2:1^2 = 4:1$

Steps in calculating area of similar triangles

Step 1: Identify the corresponding sides and determine the ratio of the corresponding sides.



$$\text{Scale factor} = \frac{\overline{ED}}{\overline{BA}} = \frac{\overline{DF}}{\overline{AC}}$$

$$= \frac{16\text{ cm}}{4\text{ cm}} = \frac{12\text{ cm}}{3\text{ cm}} = \frac{4}{1} = 4$$

Therefore the scale factor is 4. .

The ratio of the sides of ΔDEF to ΔABC is 4**:**1. This implies that the side lengths of ΔDEF are four times longer than the side lengths of ΔABC.

Step 2: Square the ratio of the sides. This will determine the ratio of the area of the two triangles. That is;

- Ratio of the area = (Ratio of the sides)^2 = (4**:**1)^2 = 16**:**1
- Ratio of area is also = (scale factor)^2 = k^2 = 4^2 = 16.

The ratio 16:1 indicates that the area of the second triangle, ΔDEF is sixteen (16) times bigger than the area of ΔABC.

Step 3: Calculate the area of ΔABC.

$$\text{Area of } \Delta\text{ABC} = \frac{1}{2} \text{ x b x h}$$

$$= \frac{1}{2}(3\text{ cm})(4\text{ cm})$$

$$= \frac{12}{2}\text{ cm}^2$$

$$= 6\text{ cm}^2$$

Therefore, the area of ΔABC is 6 cm^2.

Step 4: Multiply the area of ΔABC by the area ratio to determine the area of ΔDEF.

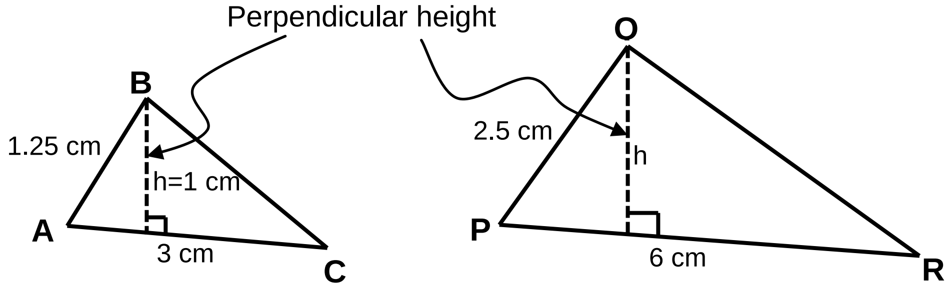
$$\text{Area of } \Delta\text{DEF} = \Delta\text{ABC x ratio of area}$$
$$= 6 \text{ cm}^2 \text{ x } 16$$
$$= 96 \text{ cm}^2$$

Therefore, the area of ΔDEF is 96 cm^2.

Now let us look at the area of similar triangles that are not right angled triangles. When calculating the area of a non-right angled triangle, use the perpendicular height. Then, follow the steps above to calculate the unknown area of the similar triangle. You learnt about perpendicular height from your Gr 7 and Gr 8 Mathematics.

Example 2

Determine the area and height of triangle PQR.



Solution:

1. Find the ratio of the corresponding sides (scale factor).

$$\text{Scale factor (k) = ratio of sides}$$
$$\text{Scale factor (k)} = \frac{\overline{PR}}{\overline{AC}} = \frac{6 \text{ cm}}{3 \text{ cm}} = 2$$

Therefore, the scale factor is 2.

2. Determine the area of triangle ABC

$$\text{Area of } \Delta\text{ABC} = \frac{1}{2}\text{bh} \quad \text{where h is the Perpendicular height}$$
$$= \frac{1}{2}(3 \text{ cm})(1 \text{ cm})$$
$$= \frac{3}{2} \text{ cm}^2$$
$$= 1.5 \text{ cm}^2$$

Therefore, area of triangle ABC is 1.5 cm^2.

3. Calculate the ratio of area.

   Ratio of area = $k^2 = 2^2 = 4$.

4. Find the area of triangle **PQR.**

   Area of Δ **PQR** = Δ **ABC** x ratio of area

   $= (1.5 \text{ cm}^2)(4)$

   $= 6 \text{ cm}^2$

   Therefore, the area of Δ PQR is 6 cm^2.

5. Now that the area is determined, find the perpendicular height of triangle PQR.

   To find the height of the triangle, rearrange the formula to make h the subject.

   That is; $A = \frac{1}{2}bh$

   $2(A) = \left[\frac{bh}{2}\right](2)$ multiply both sides by 2

   $2A = bh$

   $\frac{2A}{b} = \frac{bh}{b}$ divide both sides by b

   $h = \frac{2A}{b}$

   Now substitute the value of A and b to determine the height.

   $A = 6 \text{ cm}^2$ and $b = 6$ cm

   $h = \frac{2(6 \text{ cm}^2)}{6 \text{ cm}}$

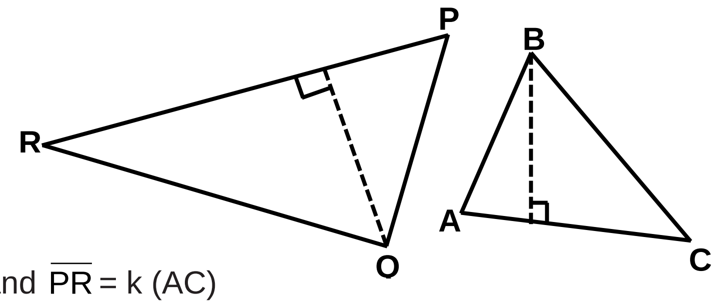
   $h = 2$ cm

   Therefore the perpendicular height of the triangle is 2 cm.

Remember the following key concepts:

- The scale factor, **k**, relates the lengths of corresponding sides of similar triangles. For example, in triangle ABC and triangle PQR.

$$\frac{\overline{PQ}}{\overline{AB}} = \frac{\overline{QR}}{\overline{BC}} = \frac{\overline{PR}}{\overline{AC}} = K$$



These relationships can also be written as follows;

$\overline{PQ} = k(\overline{AB})$, $\overline{QR} = k(BC)$ and $\overline{PR} = k\ (AC)$

- The square of the scale factor, $\mathbf{k^2}$, relates the areas of the two similar triangles.

$$\frac{\text{Area of } \Delta PQR}{\text{Area of } \Delta ABC} = k^2$$

This relationship can also be written as follows;

$$\text{Area of } \Delta PQR = k^2 (\text{ Area of } \Delta ABC)$$

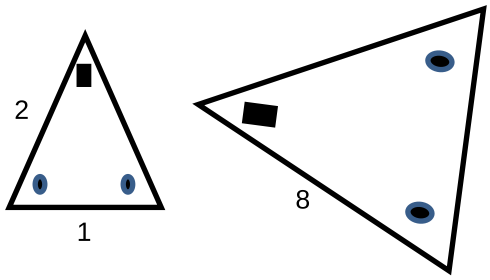
- This concept can also be applied in finding the areas of other similar figures.

NOW DO PRACTICE EXERCISES 7

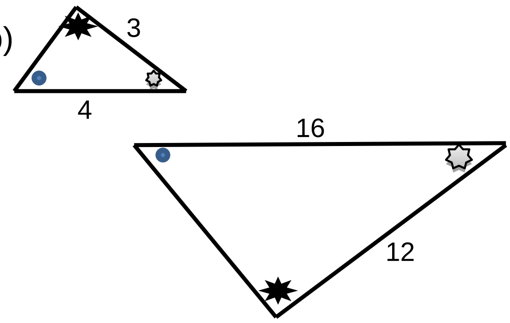
Practice Exercise 7

1. Determine the ratio of area of the following pair of similar triangles. All dimensions in cm.

(a)

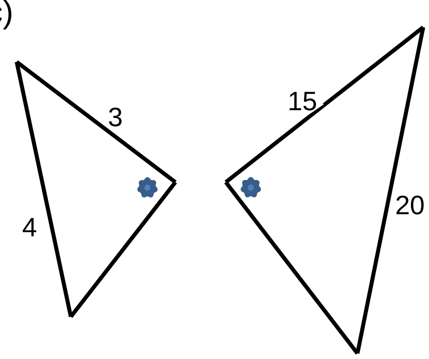


(b)

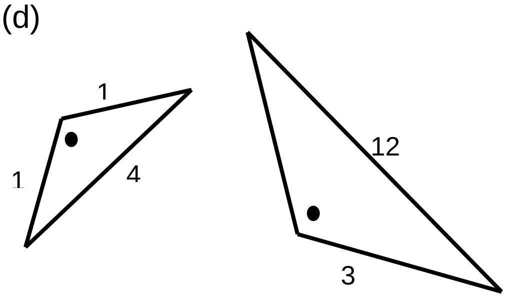


Answer:________________________ **Answer**:________________________

(c)



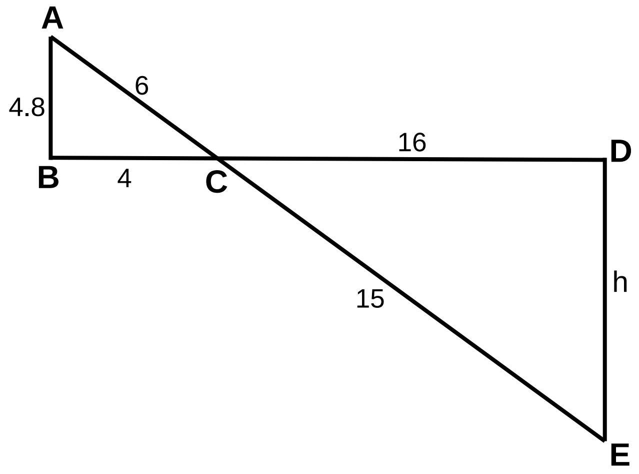
(d)



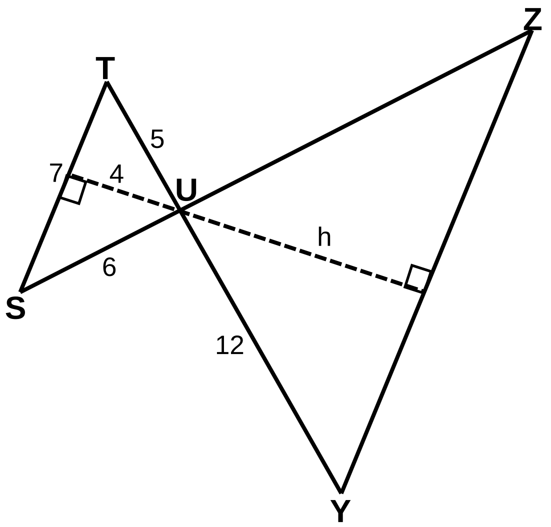
Answer:________________________ **Answer**:________________________

2. Calculate the area of all the second triangles of the following pair of similar triangles using the ratio of their areas. All dimensions are in meters, (m).

(a)



(b)



3. **Refer to the diagrams in Question 2.**

(a) Find the height of ΔCDE and ΔUYZ.

(i) Height, h, of ΔCDE

(ii) Height, h, of ΔUYZ

Answer:__________________

Answer:_____________________

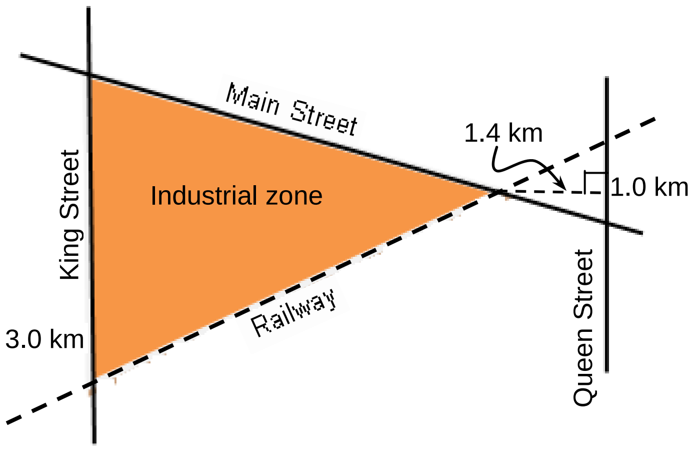
4. Problem solving:

(a) The areas of two similar triangles are 72 cm^2 and 162 cm^2. What is the ratio of the lengths of their corresponding sides?

(b) ΔABC ~ ΔXYZ and have a scale factor of 3:2. What is the ratio of their areas?

(c) ΔDEF ~ ΔPQR and have scale factor 2.5. If ΔDEF is 36 cm2, find the area of ΔPQR?

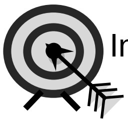
(d) The shaded region is to be an industrial zone. Find the area of the zone, assuming that King street and Queen street are parallel and that all streets and the track are straight.



CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 2

Lesson 8: Congruent Figures

In Lesson 5 of this Topic, you learnt about similar figures and their properties.



In this lesson, you will:

- define congruent figures.
- identify congruent figures and use notations to denote congruence.
- solve problems involving congruent figures.

Congruency is a branch of geometry that deals with objects which are identical in shape and size.

Congruence is the property of a plane or solid figure whereby it coincides with another plane or solid figure after it is moved, rotated, or flipped over.

Congruent figures are figures that have the same shape and size. They are basically a perfect copy of each other. The only difference may be the orientation of the figures.

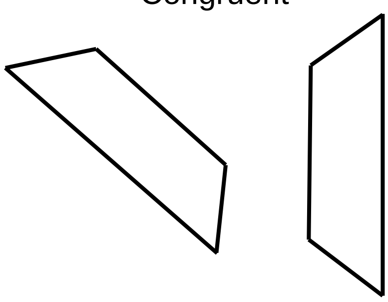
Orientation of the figure refers to the direction it is facing.

Identifying congruent figures:

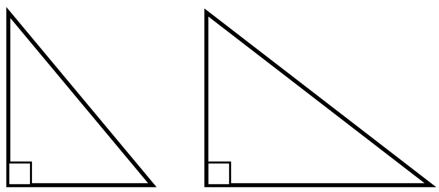
Two figures are congruent if they have the same size and shape regardless of their orientation.

Example 1

Congruent



Not Congruent



When two geometric figures are congruent, there is a correspondence between their angles and sides such that the corresponding angles are congruent (equal) and corresponding sides are congruent (equal).

Congruent means identical in shape and size.

Properties of congruent figures with straight sides:

1. Corresponding sides of congruent figures are congruent.
2. Corresponding angles of congruent figures are congruent.
3. The perimeters and areas of congruent figures are always congruent.

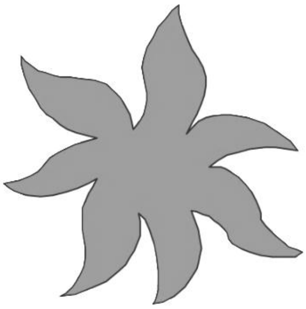
Symbol for congruence is $\cong$.

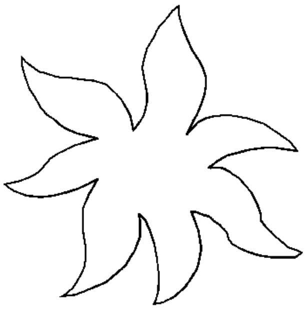
Congruent figures that have curved sides.

Figures with curves, like flower petals, ellipses and circles, can be congruent too, but these are harder to measure and compare. If either one of two curved shapes could cover the other one completely, with nothing sticking out, then those are congruent figures. All circles are similar but only circles with the same radius are congruent.

Example 2

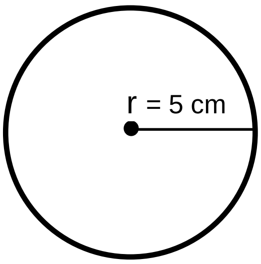
i. Leaf A Leaf B

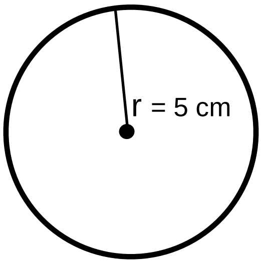




When leaf A is put over leaf B, it fits exactly. Hence, leaf A and leaf B are congruent. Leaf A $\cong$Leaf B.

ii.. Circle A Circle B



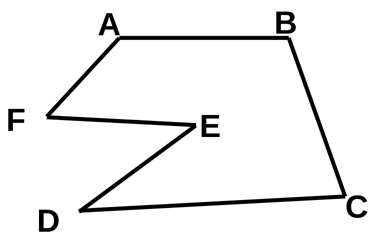


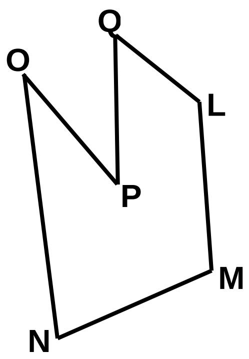
Circle A and circle B have equal radius. This implies that the two circles are congruent. This statement can also be written as circle A $\cong$ circle B

Writing congruent statements

In writing congruent statements, first identify the corresponding sides and the corresponding angles, and then use the symbol or notation, $\cong$, for congruence to write congruent statements for the congruent figures.

Example 3





Corresponding sides: Property 1

(a) $\overline{\mathbf{AB}}$ corresponds to $\overline{\mathbf{LM}}$, which means that sides **AB** and **LM** are congruent.

$\overline{\mathbf{AB}} \cong \overline{\mathbf{LM}}$

(b) $\overline{\mathbf{BC}}$ corresponds to $\overline{\mathbf{MN}}$, which means that sides BC and MN are congruent.

$\overline{\mathbf{BC}} \cong \overline{\mathbf{MN}}$

(c) $\overline{\mathbf{CD}}$ corresponds to $\overline{\mathbf{NO}}$, which means that sides CD and NO are congruent.

$\overline{\mathbf{CD}} \cong \overline{\mathbf{NO}}$

(d) $\overline{\mathbf{DE}}$ corresponds to $\overline{\mathbf{DE}}$, which means that sides DE and OP are congruent.

$\overline{\mathbf{DE}} \cong \overline{\mathbf{OP}}$

(e) $\overline{\mathbf{EF}}$ corresponds to $\overline{\mathbf{PQ}}$, which means that sides EF and PQ are congruent.

$\overline{\mathbf{EF}} \cong \overline{\mathbf{PQ}}$

(f) $\overline{\mathbf{AF}}$ corresponds to $\overline{\mathbf{LQ}}$, which means that sides AF and LQ are congruent.

$\overline{\mathbf{AF}} \cong \overline{\mathbf{LQ}}$

Corresponding angles: Property 2

(a) $\angle$**A** corresponds to $\angle$**L**, which means that $\angle$**A** and $\angle$**L** are congruent.

$\angle \mathbf{A} \cong \angle \mathbf{L}$

(b) $\angle$**B** corresponds to $\angle$**M**, which means that $\angle$**B** and $\angle$**M** are congruent.

$\angle \mathbf{B} \cong \angle \mathbf{M}$

(c) $\angle$**C** corresponds to $\angle$**N**, which means that $\angle$**C** and $\angle$**N** are congruent.

$\angle \mathbf{C} \cong \angle \mathbf{N}$

(d) $\angle$**D** corresponds to $\angle$**O**, which means that $\angle$**D** and $\angle$**O** are congruent.

$\angle \mathbf{D} \cong \angle \mathbf{O}$

(e) $\angle$**E** corresponds to $\angle$**P**, which means that $\angle$**E** and $\angle$**P** are congruent.

$\angle \mathbf{E} \cong \angle \mathbf{P}$

(f) $\angle$**F** corresponds to $\angle$**Q**, which means that $\angle$ **F** and $\angle$**Q** are congruent.

$\angle \mathbf{F} \cong \angle \mathbf{Q}$

Since the corresponding sides and the corresponding angles are congruent, the two polygons are congruent. This statement can also be written as;

Polygon **ABCDEF** $\cong$ Polygon **LMNOPQ**

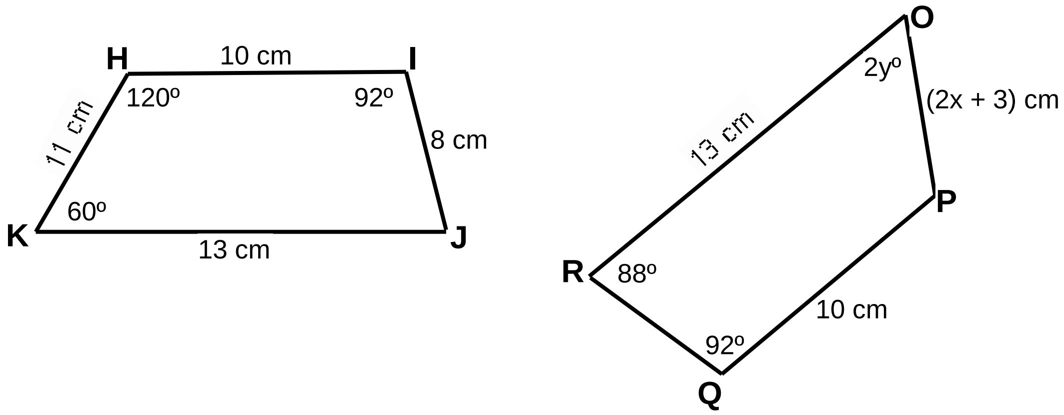
Area of the two polygons: Property 3.

If the area of polygon **ABCDEF** is equals to 76 cm^2, the area of polygon **LMNOPQ** will also be equal to 76 cm^2, since the two polygons are congruent.

Solving problems involving congruent figures:

Example 1

Look at the two trapeziums **HIJK** and **OPQR**.



(a) Determine whether they are congruent by identifying the corresponding angles and sides and showing that they equal by measurement.

(b). Determine the values of **x** and **y**.

Solution:

(a) Identify and list all corresponding sides and angles.

Corresponding sides:

i. $\overline{\mathbf{HI}}$ corresponds to $\overline{\mathbf{PQ}}$, $\overline{\mathbf{HI}} \cong \overline{\mathbf{PQ}}$ = 10 cm

ii. $\overline{\mathbf{JK}}$ corresponds to $\overline{\mathbf{RO}}$, $\overline{\mathbf{JK}} \cong \overline{\mathbf{RO}}$ = 13 cm

iii. $\overline{\mathbf{IJ}}$ corresponds to $\overline{\mathbf{RQ}}$, $\overline{\mathbf{IJ}} \cong \overline{\mathbf{RQ}}$ = 8cm

iv. $\overline{\mathbf{HK}}$ corresponds to $\overline{\mathbf{PO}}$, $\overline{\mathbf{HK}} \cong \overline{\mathbf{PO}}$ = 11 cm = (2x + 3) cm

Corresponding angles:

i. $\angle\mathbf{H} \cong \angle\mathbf{P} = 120^\circ$

ii. $\angle\mathbf{I} \cong \angle\mathbf{Q} = 92^\circ$

iii. $\angle\mathbf{J} \cong \angle\mathbf{R} = 88^\circ$

iv. $\angle\mathbf{K} \cong \angle\mathbf{O} = 60^\circ = 2y^\circ$

(b) Determine the value of **x** and **y**.

i. Determine the value of x:

The side, **PO** = (2x + 3) is correspondent to **HK** = 11 cm. Using property 1 of congruent figures, find the value of x by equating the corresponding sides together.

That is; $\overline{PO} = \overline{HK}$

$$(2x + 3) = 11$$

$$2x = 11 - 3 \quad \text{(Subtract 3 from both sides)}$$

$$\frac{2x}{2} = \frac{8}{2} \quad \text{(Divide both sides by 2)}$$

$$x = 4$$

Therefore, **x** is 4 cm and the length of side $\overline{PO}$ = 11 cm.

ii. Determine the value of **y**:

$\angle$**O** = $(2y)^o$ is correspondent to $\angle$**K** = 60^o. Using property 2 of congruent figures, determine the value of y by equating the angles together.

That is; $\angle O = \angle K$

$$2y = 60^o$$

$$y = 30^o \quad \text{Dividing both sides by 2}$$

Therefore, y = 30^0 and the size of $\angle$O = 60^0.

Example 2

Circle 1 and Circle 2 are congruent. If the circumference of Circle 1 is 68.2 cm, find the radius of the Circle 2.

Solution: Since the circumference of Circle 1 is given, find its radius.

C = 68.2 cm, π = 3.14.

$$C = 2\pi r$$

$$6.28 = 2 \times 3.14 \times r$$

$$6.28 = 6.28 \times r$$

$$\frac{6.28}{6.28} = r$$

$$10 = r \text{ or } r = 10 \text{ cm}$$

Therefore, the radius of Circle 1 is 10 cm.

Since the circles are congruent, their radii will also be congruent. This means that the radius of circle 1 is equal to the radius of circle 2.

That is;

Radius of Circle 1 = Radius of Circle 2 = 10 cm

Therefore, the radius of Circle 2 is 10 cm.

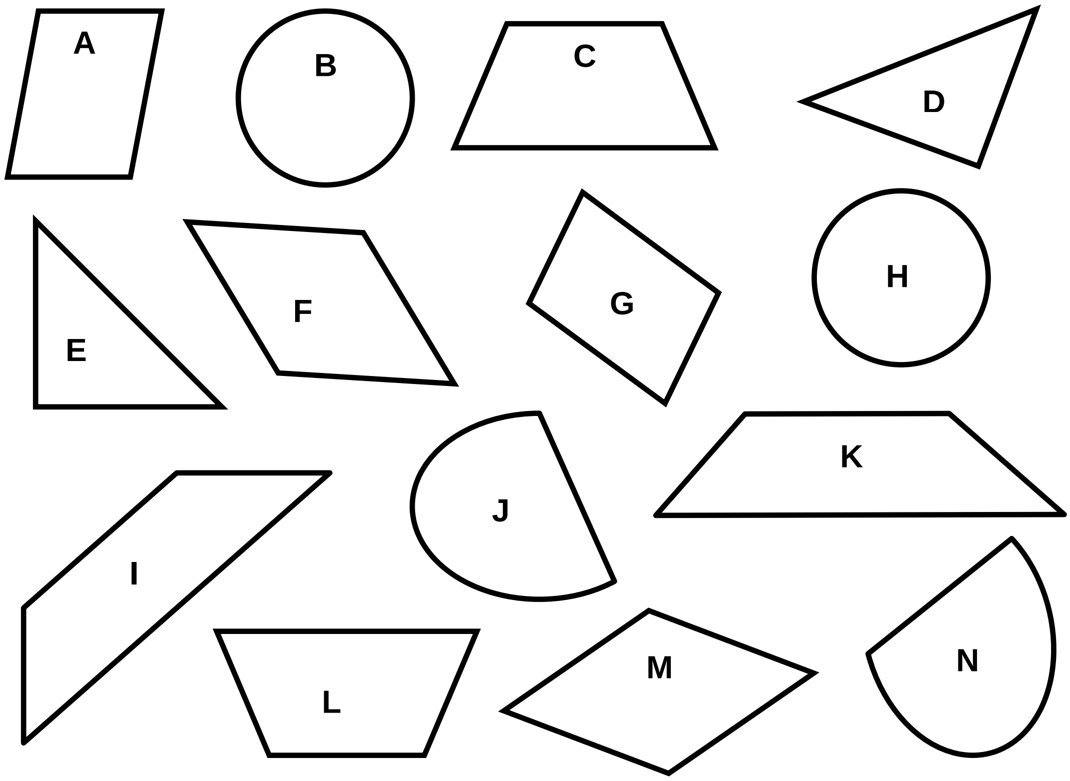
Main Points

- Congruent figures are the same shapes and sizes.
- Corresponding sides and angles of congruent figures are always equal.
- All congruent figures are similar but not all similar figures are congruent.

NOW DO PRACTICE EXERCISES 8

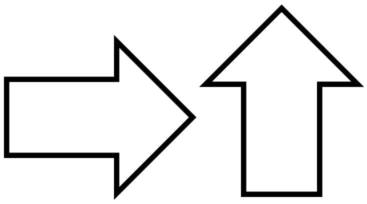
Practice Exercise 8

1. Identify and select the pair of congruent figures from the following figure. Use the congruence symbol to denote the pair of congruent figures.

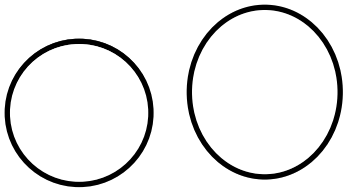


2. Determine whether the following pairs of figures are congruent or not. Give an explanation for your answer.

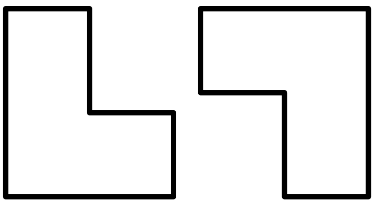
a.



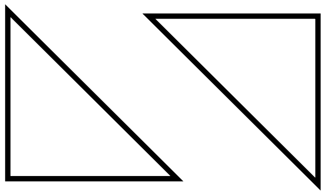
b.



b.

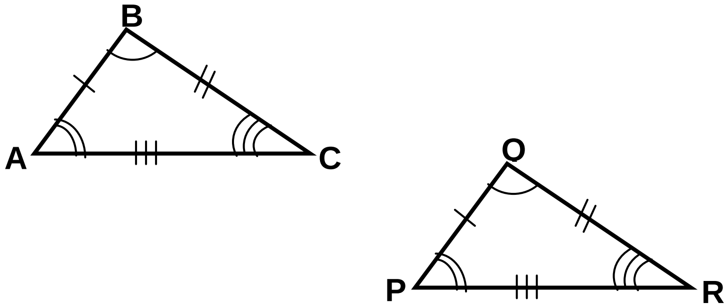


d.

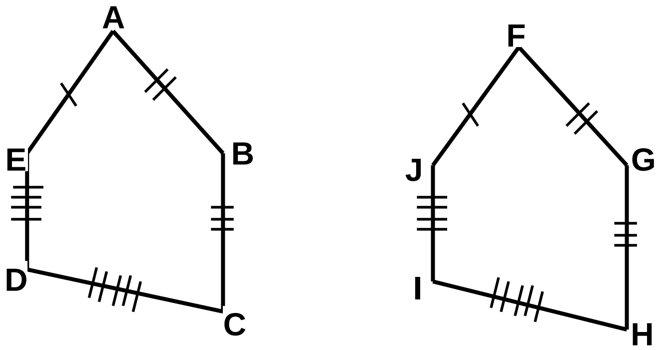


3. Identify and list all the corresponding sides and angles of the following pairs of figures. Then write the congruence statement for the two figures.

(a)



(b)



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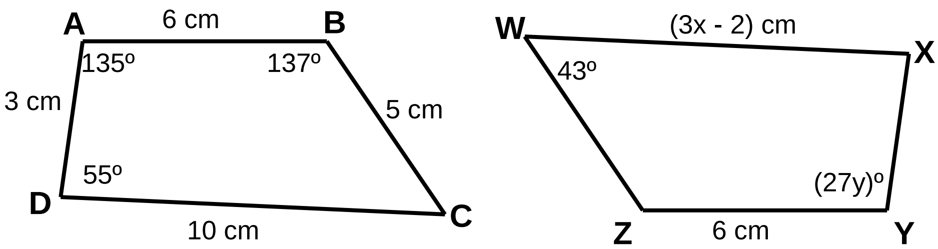
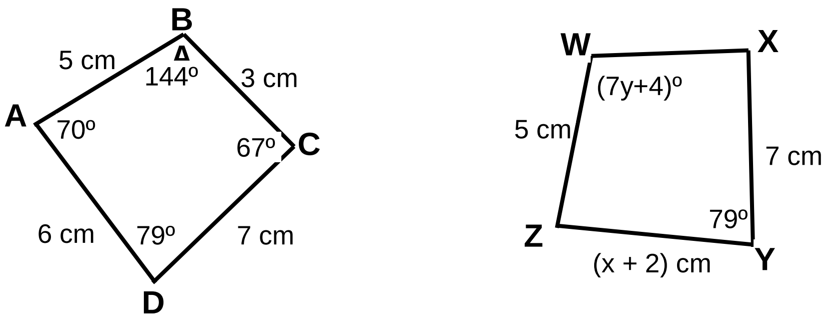
4. Find the values of **x** and **y** in the following in the following pair of figures.

(a)



(b)



5. Refer to Question 4.

i. Match the sides and angles in Question 4(a).
.
Corresponding sides: Corresponding angles:

ii. Are the figures congruent? Justify your answer. Write a congruent statement for the two figures.

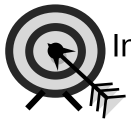
iii. Match the sides and angles in Question 4(b)
.
Corresponding sides: Corresponding angles:

iv. Are the figures congruent? Justify your answer. Write a congruent statement for the two figures.

CHECK YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 2.

Lesson 9: Congruent Triangles

In the last lesson, you learnt about congruent figures and their properties.



In this lesson, you will:

- define congruent triangles.
- identify congruent triangles and use notations to denote congruence.
- solve problems involving congruent triangles.

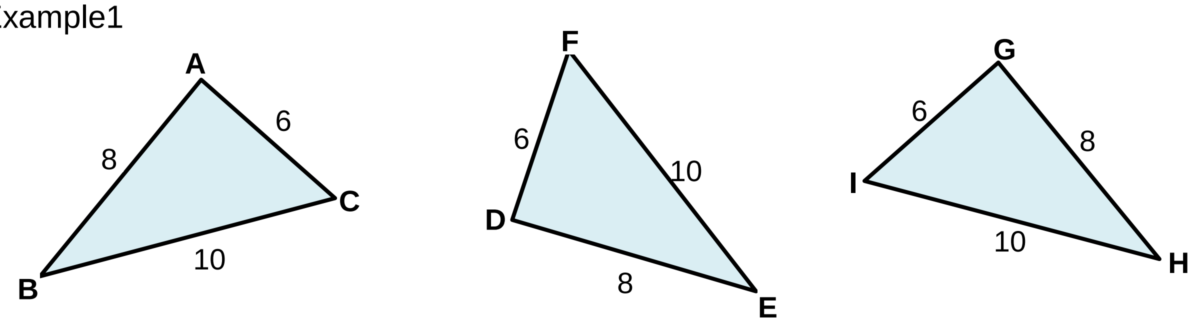
You learnt about similar triangles in Lesson 6. Now you will learn about congruent triangles.

Congruent triangles are triangles that have the same size and shape. This means that the corresponding sides are equal and the corresponding angles are equal.

The equal sides and angles of congruent triangles may not be in the same position if there is a turn or a flip, but they will be the same..

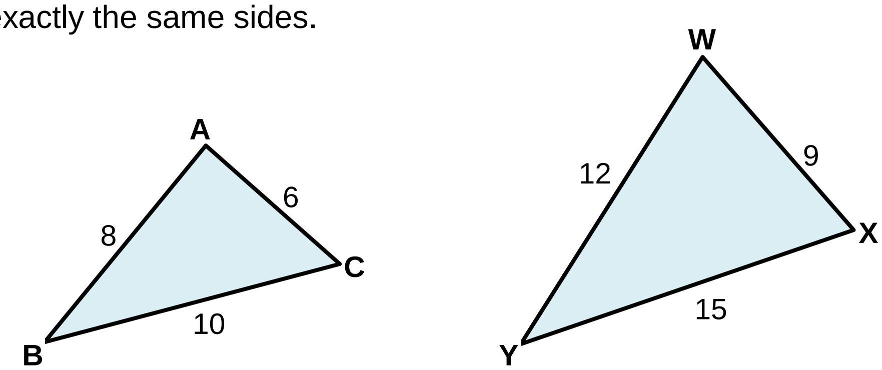
If the sides of the triangles are the same, then the triangles are congruent.

Example1



Triangle **ABC** is congruent to Triangle **DEF** and Triangle **GHI** because they all have exactly the same sides.

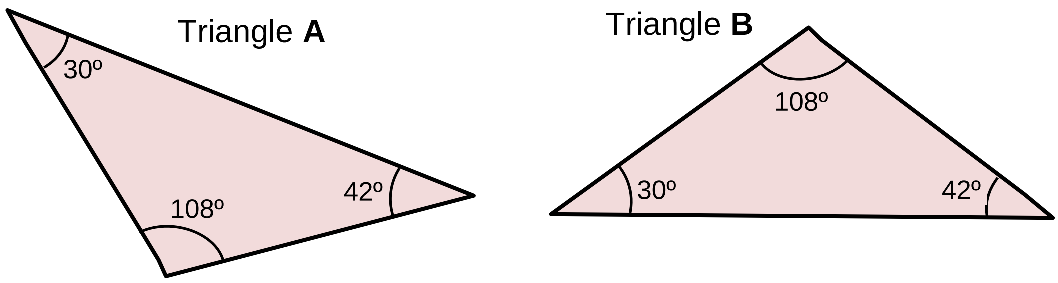
Example 2



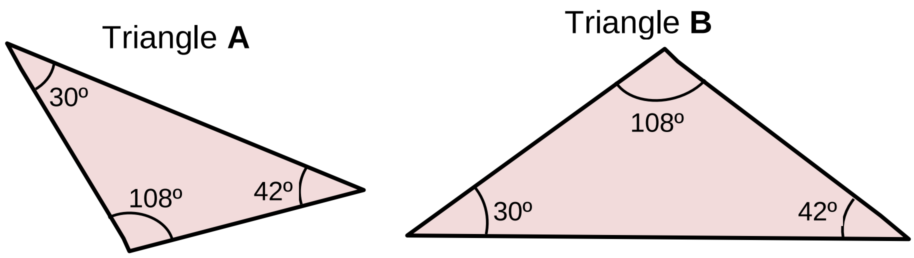
Triangle ABC is not congruent to Triangle WXY because they **do not** have **exactly the same corresponding sides.**

But this does not work always with angles.

For example, these two triangles with the same angles might be congruent.



Triangle **A** and Triangle **B** are **congruent** only because they have the **same size**. They might **NOT** be congruent if they are of **different sizes**.

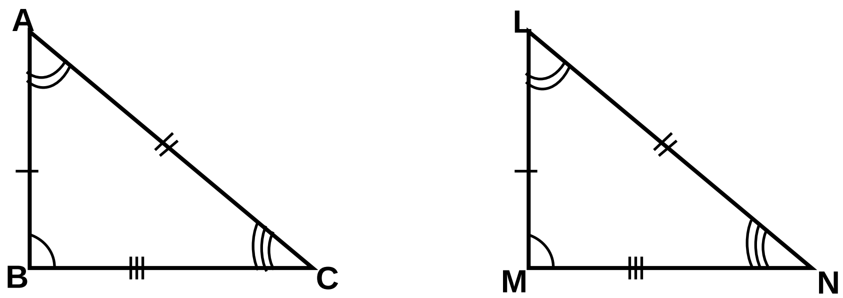


Triangle **A** and Triangle **B** are not congruent because even though the angles match, one is larger than the other.

In the diagram below, the two triangles **ABC** and **LMN** are **<u>congruent</u>** because every corresponding side has the **same length**, and every corresponding angle has the **same measure**. The angle at **A** has the same measure (in degrees) as the angle at **L**, the side **AB** is the same length as the side **LM,** etc.

Two triangles are congruent if they are the same in shape and size. This means that all three corresponding sides and corresponding angles are equal.

If two triangles are congruent, we often mark corresponding sides and angles like this:



The sides marked with one line are equal in length. Similarly for the sides marked with two lines and three lines.

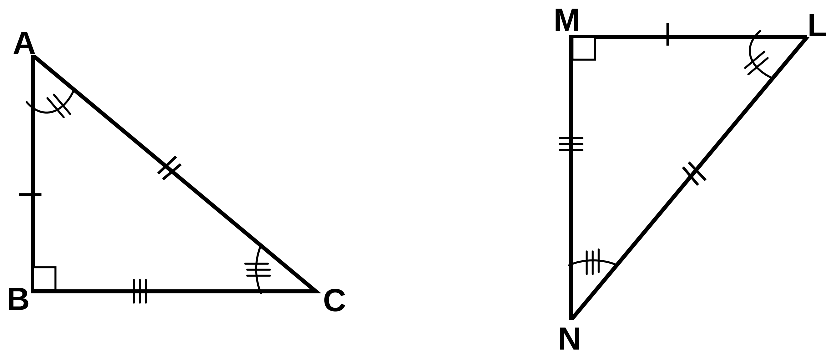
The angles marked with one arc are equal in size. Similarly for the angles marked with two arcs or three arcs.

For instance, if $\Delta\mathbf{ABC} \cong \Delta\mathbf{LMN}$, then the following relations will result.

Corresponding sides: $\overline{\mathbf{AB}} = \overline{\mathbf{LM}}$, $\overline{\mathbf{BC}} = \overline{\mathbf{MN}}$, $\overline{\mathbf{AC}} = \overline{\mathbf{LN}}$ and

Corresponding angles: $\angle\mathbf{A} = \angle\mathbf{L}$, $\angle\mathbf{B} = \angle\mathbf{M}$, $\angle\mathbf{C} = \angle\mathbf{N}$

In the diagram on the previous page, the triangles are drawn next to each other and it is obvious they are identical. However, one triangle may be rotated, flipped over (reflected), or the two triangles may share a common side



To be congruent two triangles must be of the same shape and size. However one triangle can be rotated, and as long as they are identical, the triangles are still congruent. In the figure above, the triangle LMN is congruent to ABC even though it rotated anti-clockwise about 90°.

Remember the Properties of Congruent Triangles

> **If two triangles are congruent, then each part of one triangle (side and angle) is congruent to the corresponding part in the other triangle.**

This is the true value of the concept. Once you have proven two triangles are congruent, you can find the angles or sides of one of them from the other.

It is helpful to remember this important concept by using the acronym **CPCTC which stands for "Corresponding Parts of Congruent Triangles are Congruent".**

How can we tell if triangles are congruent?

Any triangle is defined by six measures (three sides, three angles). But you don't need to know all of them to show that two triangles are congruent. Various groups of three will do.

We can tell whether two triangles are congruent without testing all the sides and all angles of the two triangles. There are four Conditions or Rules to check for two triangles to be congruent. These are:

1. **SSS Congruence Rule:** (Side-Side-Side)

   All three corresponding sides are equal in length.

2. **SAS Congruence Rule:** (Side-Angle-Side)

   A pair of corresponding sides and the included angle are equal.

3. **ASA Congruence Rule:** (Angle-Side-Angle)
   A pair of corresponding angles and the included side are equal.

4. **AAS Congruence Rule:** (Angle-Angle-Side)
   A pair of corresponding angles and a non-included side are equal.

There is also another rule for right triangles. This is:

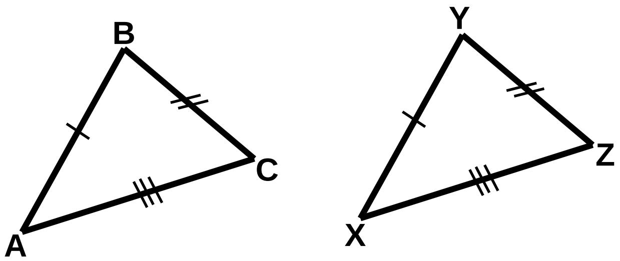
5. **HL Congruence Rule:** (Hypotenuse-Leg of a right triangle)
   Two right triangles are congruent if the hypotenuse and one leg are equal.

Let us look at each rule to better understand them.

The SSS Congruence Rule

The Side-Side-Side rule states that if three sides of one triangle are equal to the corresponding three sides of another triangle, then the two triangles are congruent.

For example: The diagram shows two triangles: ΔABC and ΔXYZ



Three sides of ΔABC corresponds to the three sides of ΔXYZ

$\overline{AB}$ corresponds to $\overline{XY}$

$\overline{BC}$ corresponds to $\overline{YZ}$

$\overline{AC}$ corresponds to $\overline{XZ}$

Since the corresponding sides of the two triangles are marked equal, we have:

$\overline{AB} = \overline{XY}$

$\overline{BC} = \overline{YZ}$

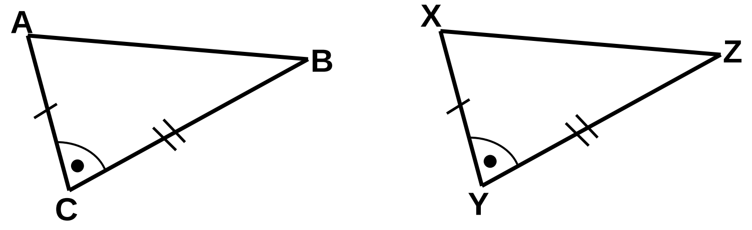
$\overline{AC} = \overline{XZ}$

Therefore, $\Delta ABC \cong \Delta XYZ$

The SAS Congruence Rule

The Side-Angle-Side rule states that if two sides and the included angle of one triangle are equal to the corresponding two sides and the included angle of another triangle, then the two triangles are congruent.

For example: The diagram shows two triangles: ΔABC and ΔXYZ .



Two sides and their included angle in ΔABC correspond to the two sides and their included angle in ΔXYZ .

$\overline{AC}$ corresponds to $\overline{XY}$

$\overline{CB}$ corresponds to $\overline{YZ}$

$\angle C$ corresponds to $\angle Y$

Since the corresponding sides and their including angle in the two triangles are marked equal, we have:

$\overline{AB} = \overline{XZ}$

$\overline{BC} = \overline{YZ}$

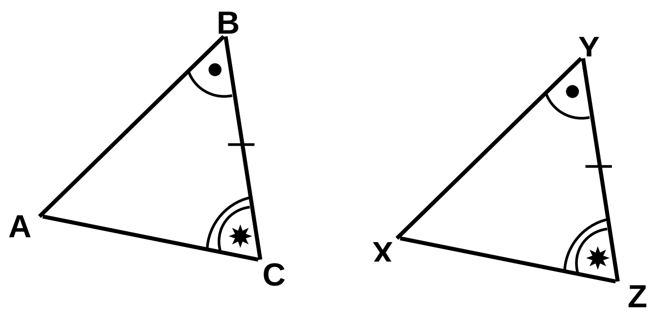
$\angle C = \angle Y$

Therefore, $\Delta ABC \cong \Delta XYZ$.

The ASA Congruence Rule

The Angle-Side- Angle rule states that if two angle and the included side of one triangle are equal to the corresponding two angles and the included side of another triangle, then the two triangles are congruent.

For example: The diagram shows two triangles: ΔABC and ΔXYZ .



Two angles and their included side in ΔABC corresponds to the two angles and their included side in ΔXYZ .

$\angle B$ corresponds to $\angle Y$

$\angle C$ corresponds to $\angle Z$

$\overline{BC}$ corresponds to $\overline{YZ}$

Since the corresponding angles and their including sides in the two triangles are marked equal, we have:

$\angle B = \angle Y$

$\angle C = \angle Z$

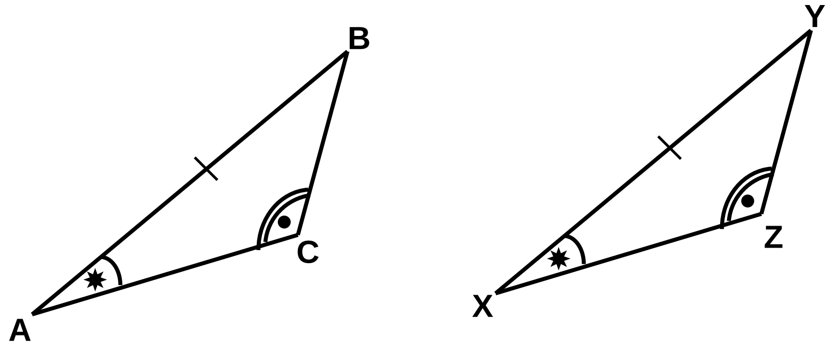
$\overline{BC} = \overline{YZ}$

Therefore, $\Delta ABC \cong \Delta XYZ$.

The AAS Congruence Rule

The Angle-Angle-Side rule states that if two angles and a non-included side of one triangle are equal to two angles and a non-included side of another triangle, then the two triangles are congruent.

For example: The diagram shows two triangles: ΔABC and ΔXYZ .



Two angles and a side in ΔABC correspond to the two angles and a side in ΔXYZ

$\angle A$ corresponds to $\angle X$

$\angle C$ corresponds to $\angle Z$

$\overline{AB}$ corresponds to $\overline{XY}$

Since the corresponding angles and a side in the two triangles are marked equal, we have:

$\angle A = \angle X$

$\angle C = \angle Z$

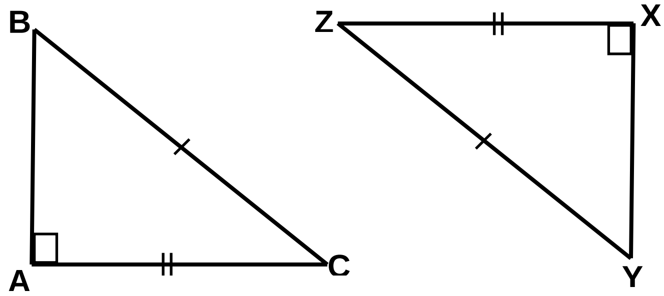
$\overline{AB} = \overline{XY}$

Therefore, $\Delta ABC \cong \Delta XYZ$.

The AAS Congruency Rule may sometimes be referred to as **SAA Rule.**

The Right Triangle or HL Congruence Rule

The Hypotenuse-Leg rule states that if the hypotenuse and a leg of a right triangle are equal to the corresponding hypotenuse and a leg of another right triangle, then the two triangles are congruent.



The hypotenuse and a side in $\triangle ABC$ correspond to the hypotenuse and a side in $\triangle XYZ$

$\overline{BC}$ corresponds to $\overline{YZ}$

$\overline{AC}$ corresponds to $\overline{XZ}$

Since the corresponding hypotenuse and a side in the two triangles are marked equal, we have:

$\overline{BC} = \overline{YZ}$

$\overline{AC} = \overline{XZ}$

Therefore, $\triangle ABC \cong \triangle XYZ$

Note:

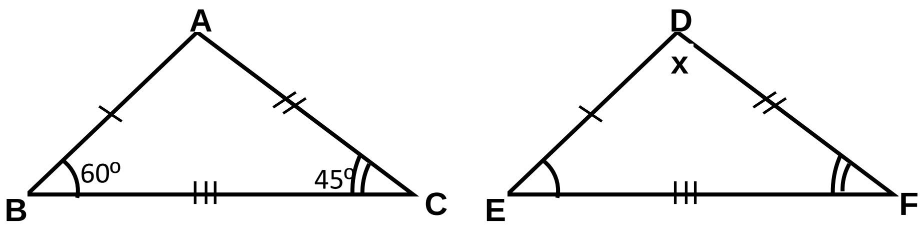
- For the ASA rule the given side must be included and for AAS rule the side given must not be included. The trick is we must use the same rule for both the triangles that we are comparing.

- If all the corresponding angles of two triangles are the same, the triangles will be the same shape, but not necessarily the same size.
  They are called similar triangles but not congruent triangles.

Now go to the next page and study the solved example.

Here is the example.

Triangle ABC is congruent to triangle DEF as shown. Find **x.**



Solution:

Given: $\Delta ABC \cong \Delta DEF$

You learnt that $m\angle A + m\angle B + m\angle C = 180º$ (Angle sum of triangles is 180º)

Hence we have, $m\angle A = 180º – (m\angle B + m\angle C)$

Substitute 60º for $m\angle B$ and 45º for $m\angle C$)

$m\angle A = 180º – (60º + 45º)$

$= 180º – 105º$

$\boldsymbol{m\angle A = 75º}$

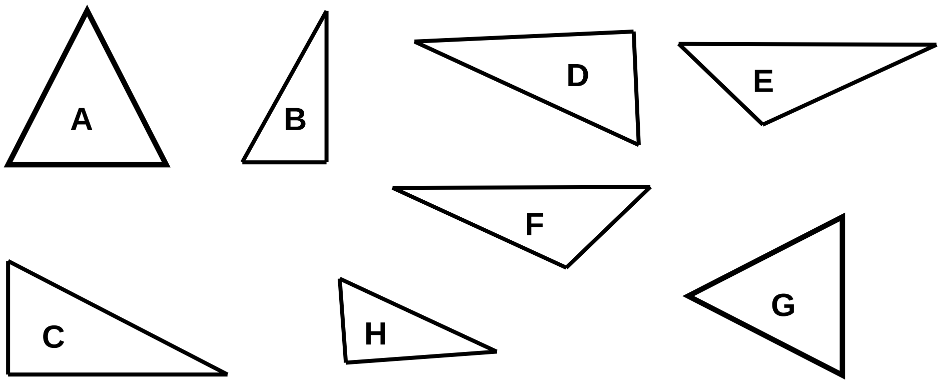
Since $\Delta ABC \cong \Delta DEF$, then $m\angle D = m\angle A$

Substitute x for $m\angle D$ and 75º for $m\angle A$, we have **x = 75º.**

NOW DO PRACTICE EXERCISES 9

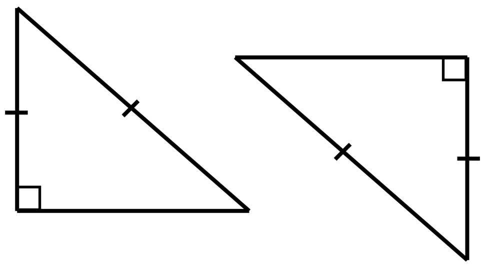
Practice Exercise 9

1. Identify the pair of congruent triangles and list them down:



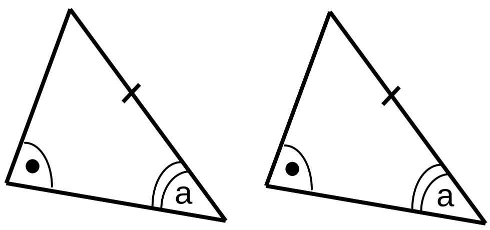
2. State whether the following pair of triangles are congruent or not. Give a reason for your answer.

(a)



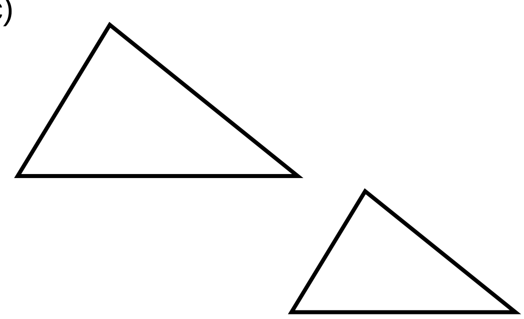
Congruent: ____________________
Reason: ______________________
____
____
____

(b)



Congruent:____________________
Reason:______________________
____
____

(c)



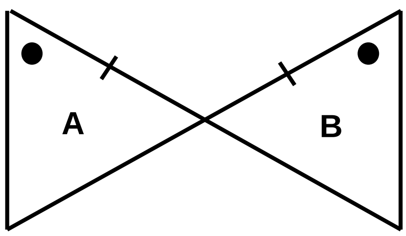
Congruent:
Reason:______________________
____
____

3. For the following pair of triangles:

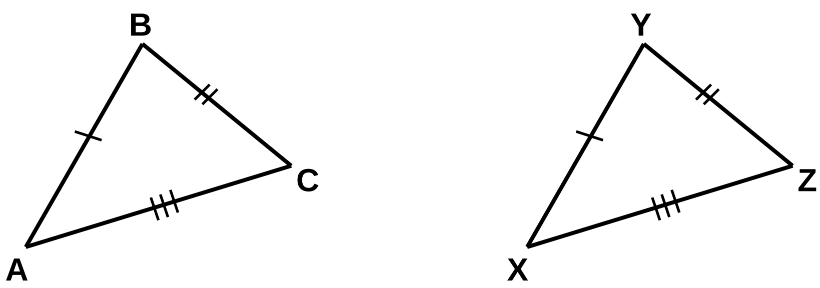
   i. Determine whether the triangles are congruent.

   (ii) If the triangles are congruent, what else can be said about them? Write a congruence statement of two triangles.

(a)

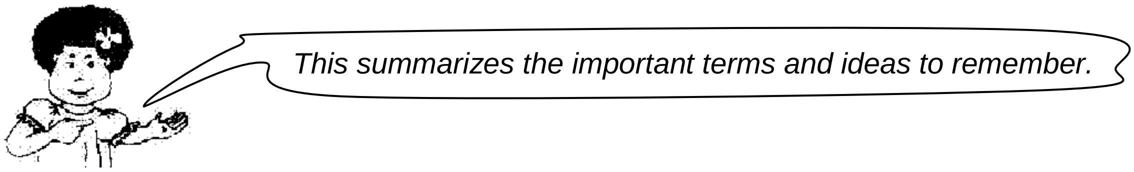


(b)



CHECK YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 2

TOPIC 2: SUMMARY



- **Similar figures** are figures that are the same but of different sizes.
- Properties of similar figures
  - ❖ The corresponding angles of similar figures are equal.
  - ❖ The lengths of the corresponding sides of similar figures are in the same ratio.
  - ❖ The value of this ratio is equal to the scale factor, *k*.
- The notation used to denote similarity is the symbol ($\sim$).
- **The reducing factor** is the ratio by which the corresponding sides are decreased.
- **The enlargement factor** is the ratio by which the corresponding sides are increased.
- **Similar triangles** are triangles that are the same but different in sizes.
- **Two triangles are similar if:**
  - ❖ Two pairs of corresponding sides are in the same ratio and the angles included between them are equal.
  - ❖ The corresponding sides are in the same ratio
  - ❖ The corresponding angles are the same and equal..
- The **ratio of the areas of two similar triangles** is equal to the **ratio of the squares** of any two corresponding sides.
- **Congruence** is the property of a plane or solid figure whereby it coincide with another plane or solid figure after it is moved, rotated, or flipped over.
- Two triangles are **congruent** if they have exactly the **same three sides** and exactly the **same three angles**. This means that **all corresponding sides and angles** are congruent and one may be the mirror image of the other.
- **Corresponding angles** are angles in the same position.
- **Corresponding sides** are sides that are in the same position.
- **An included side** is a side between two angles.
- **An included angle** is an angle between two sides.
- **Rules to check for congruent triangles:**
  - ❖ **SSS Congruence Rule:** (Side-Side-Side)**.** All three corresponding sides are equal in length.
  - ❖ **SAS Congruence Rule:** (Side-Angle-Side). A pair of corresponding sides and the included angle are equal.
  - ❖ **ASA Congruence Rule:** (Angle-Side-Angle). A pair of corresponding angles and the included side are equal.
  - ❖ **AAS Congruence Rule:** (Angle-Angle-Side). A pair of corresponding angles and a non-included side are equal
  - ❖ **HL Congruence Rule:** (Hypotenuse-Leg of a right triangle).Two right triangles are congruent if the hypotenuse and one leg are equal.

REVISE LESSONS 5- 9. THEN DO TOPIC TEST 2 IN ASSIGNMENT 6

ANSWERS TO PRACTICE EXERCISES 5 TO 9.

Practice Exercise 5

1. A and S F and Q
   B and P G and M
   C and K H and R
   D and O I and T
   E and N J and L

2. (a) i. Trapezium LTMN ii. Trapezium DABC

   (i) $\angle T = 90^o$ $\angle D = 90^0$

   (ii) $\angle L = 90^o$ $\angle A = 90^0$

   (iii) $\angle N = 90^o + 45^o = 135^o$ $\angle B = 135^0$

   (iv) $\angle M = 45^o$ $\angle C = 45^0$

   (b)

   (i) $\angle L$ corresponds to $\angle D$, $\angle L = \angle D = 90^0$

   (ii) $\angle T$ corresponds to $\angle A$, $\angle T = \angle A = 90^0$

   (iii) $\angle N$ corresponds to $\angle B$, $\angle N = \angle B = 135^0$

   (iv $\angle M$ corresponds to $\angle C$,) $\angle M = \angle C = 45^0$

   (c).

ABCD	LTMN	Ratio of the sides
$\overline{AB}$ = 6 units	$\overline{TN}$ = 2 units	$\dfrac{\overline{AB}}{\overline{TN}} = \dfrac{6}{2} = 3$
$\overline{BC}$ = 8.4 units	$\overline{NM}$ = 2.8 units	$\dfrac{\overline{BC}}{\overline{NM}} = \dfrac{8.4}{2.8} = 3$
$\overline{CD}$ = 12 units	$\overline{ML}$ = 4 units	$\dfrac{\overline{CD}}{\overline{ML}} = \dfrac{12}{4} = 3$
$\overline{AD}$ = 6 units	$\overline{TL}$ = 2 units	$\dfrac{\overline{AD}}{\overline{TL}} = \dfrac{6}{2} = 3$

   (d) corresponding angles are equal.

   (e) enlargement factor = 3.

   (f) Trapezium TNML $\sim$ trapezium ABCD.

3. (a) All angles are at 90^o.

(b)

TRNP	EFGD	Ratio of the sides
$\overline{TR}$ = 8 units	$\overline{EF}$ = 6 units	$\frac{\overline{TR}}{\overline{EF}} = \frac{8}{6} = \frac{4}{3}$
$\overline{RN}$ = 6 units	$\overline{FG}$ = 4 units	$\frac{\overline{RN}}{\overline{FG}} = \frac{6}{4} = \frac{3}{2}$
$\overline{NP}$ = 8 units	$\overline{GD}$ = 6 units	$\frac{\overline{GD}}{\overline{NP}} = \frac{8}{6} = \frac{4}{3}$
$\overline{PT}$ = 6 units	$\overline{ED}$ = 4 units	$\frac{\overline{PT}}{\overline{ED}} = \frac{6}{4} = \frac{3}{2}$

(c) Figures not similar because their corresponding sides are not in proportion. That is; $\frac{4}{3} \neq \frac{3}{2}$.

Practice Exercise 6

1. (a)

Similar triangles	Explanation	Corresponding sides
$\Delta ABC \sim \Delta ADE$	• Corresponding angles are equal. • Ratios of corresponding sides are in proportion.	AB corresponds to AD BC corresponds to DE CA corresponds to EA

Scale factor = 3.

$\frac{\overline{AC}}{\overline{AE}} = 3, \frac{15}{x} = 3$

$15 = 3x$

$\therefore x = \frac{15}{3} = 5 \text{ cm}$

$\frac{\overline{BC}}{\overline{DE}} = 3, \quad \frac{10}{y} = 3$

$3y = 10$

$\therefore y = \frac{10}{3} = 3\frac{1}{3} \text{ cm or } 3.33 \text{ cm}$

(b)

Similar triangles	Explanation	Corresponding sides
$\Delta AEB \sim \Delta ACD$	• Corresponding angles are equal. • Ratios of corresponding sides are in proportion.	AE corresponds to AC EB corresponds to CD BA corresponds to AD

Scale factor = 2.

$x = \frac{\overline{AD}}{k}$

$x = \frac{(6+4)}{2} = \frac{10}{2} = 5$

$\therefore x = 5$ cm

$\overline{AC} = k(\overline{AE})$

$5 + y = 2(4)$

$y = 8 - 5$

$y = 3$

$\therefore y = 3$ cm

2. $\Delta AEB \sim \Delta ACD$

ET corresponds to RB, TC corresponds to BC and EC corresponds to RC.

$$\frac{\overline{EC}}{\overline{RC}} = \frac{\overline{TE}}{\overline{BR}}$$

$$\frac{6.5}{4.2} = \frac{\overline{TE}}{1.2}$$

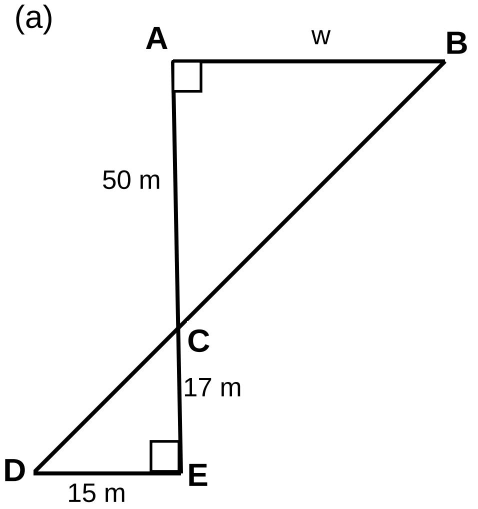
$$6.5 \times 1.2 = 4.2 \text{ x } \overline{TE}$$

$$\frac{7.8}{4.2} = \overline{TE}$$

$$\therefore \overline{TE} = 1.86\,\text{m}$$

Therefore Height of the tree is 1.86 metres

3. (a)



(b) **Corresponding sides**

$\overline{AB}$ corresponds to $\overline{ED}$

$\overline{BC}$ corresponds to $\overline{DC}$

$\overline{CA}$ corresponds to $\overline{CE}$

Width of the river = AB

$$\frac{\overline{AB}}{\overline{ED}} = \frac{\overline{CA}}{\overline{CE}}$$

$$\frac{w}{15} = \frac{50}{17}$$

$$w \times 17 = 50 \times 15$$

$$17w = 750$$

$$w = 44.12\,\text{m}$$

Therefore, Width of the river is 44 m.

Practice Exercise 7

1. (a) BigΔ :SmallΔ **4 : 1** (b) BigΔ :SmallΔ **4 : 1** (c) BigΔ :SmallΔ **5 : 1** (d). BigΔ :SmallΔ **3 : 1**

2. (a). i. $A_{\Delta ABC} = 9.6\,\text{m}^2$, ratio of area = 1: 16.

ii.

$$A_{\Delta EDC} = A_{\Delta ABC} \times \text{ratio of area}$$

$$= 9.6\,\text{m}^2 \times 16$$

$$\therefore A_{\Delta EDC} = 153.6\,\text{m}^2$$

(b). i. $A_{\Delta STU} = 14\,\text{m}^2$, ratio of area = 1: 5.76.

ii.

$$A_{\Delta ZYU} = A_{\Delta STU} \times 5.76$$

$$= 14\,\text{m}^2 \times 5.76$$

$$\therefore A_{\Delta ZYU} = 80.64\,\text{m}^2$$

3. (a) i. Height of ΔEDC = $\frac{2A}{b}$ = $\frac{2 \times 153.6\ m^2}{16\ m}$ = **19.2 m**

Ii. Height of ΔUYZ = $\frac{2A}{b}$ = $\frac{2 \times 80.64\ m^2}{16.8\ m}$ = **9.6 m**

4. (a) 2 : 3 (b) 9 : 4

(c) Area of ΔPQR = Area of ΔDEF x k^2 = 36 m^3 x $(2.5)^2$ = **225 m^2**

(d). scale factor = 3

Area of Small triangle = 0.7 km^2

Area of the zone = 6.3 km^2

Practice Exercise 8

1. A and G, B and H, C and L, D and E, F and M, J and N, I and K .

2. (a). **Congruent**: when the second shape is rotated 900 clockwise, the shapes will fit into each other exactly.

(b) **Not congruent**: One circle is bigger than the other.

(c) **Congruent**: when the second shape is rotated 1800 either clockwise or anticlockwise, the shapes will fit exactly into each other.

(d) **Congruent**: when the second shape is rotated 1800 either clockwise or anticlockwise, the shapes will fit exactly into each other.

3. (a)

Corresponding sides	Corresponding angles
$\overline{AB}$ corresponds to $\overline{PQ}$, $\overline{AB} = \overline{PQ}$ $\overline{BC}$ corresponds to $\overline{QR}$, $\overline{BC} = \overline{QR}$ $\overline{CA}$ corresponds to $\overline{RP}$, $\overline{CA} = \overline{RP}$	$\angle A = \angle P$ $\angle B = \angle Q$ $\angle C = \angle R$

Therefore, $\Delta ABC \cong \Delta PQR$

(b)

Corresponding sides	Corresponding angles
$\overline{AB}$ corresponds to $\overline{FG}$, $\overline{AB} = \overline{FG}$ $\overline{BC}$ corresponds to $\overline{GH}$, $\overline{BC} = \overline{GH}$ $\overline{CD}$ corresponds to $\overline{HI}$, $\overline{CD} = \overline{HI}$ $\overline{DE}$ corresponds to $\overline{IJ}$, $\overline{DE} = \overline{IJ}$ $\overline{EA}$ corresponds to $\overline{JF}$, $\overline{EA} = \overline{JF}$	$\angle A = \angle F$ $\angle B = \angle G$ $\angle C = \angle H$ $\angle D = \angle I$ $\angle E = \angle J$

Therefore, Pentagon ABCDE $\cong$ Pentagon FGHIJ

4. (a) x = 4 y = 5

(b) x = 4 y = 20

5. i.

Corresponding sides	Corresponding angles
$\overline{AB}$ corresponds to $\overline{YZ}$, $\overline{AB} = \overline{YZ}$ $\overline{BC}$ corresponds to $\overline{ZW}$, $\overline{BC} = \overline{ZW}$ $\overline{CD}$ corresponds to $\overline{WX}$, $\overline{CD} = \overline{WX}$ $\overline{DA}$ corresponds to $\overline{XY}$, $\overline{DA} = \overline{XY}$	$\angle A = \angle Y$ $\angle B = \angle Z$ $\angle C = \angle W$ $\angle D = \angle X$

ii. Yes. Corresponding sides and angles are equal.
trapezium ABCD $\cong$ trapezium YZWX

iii.

Corresponding sides	Corresponding angles
$\overline{AB}$ corresponds to $\overline{ZW}$, $\overline{AB} = \overline{ZW}$ $\overline{BC}$ corresponds to $\overline{WX}$, $\overline{BC} = \overline{WX}$ $\overline{CD}$ corresponds to $\overline{XY}$, $\overline{CD} = \overline{XY}$ $\overline{DA}$ corresponds to $\overline{YZ}$, $\overline{DA} = \overline{YZ}$	$\angle A = \angle Z$ $\angle B = \angle W$ $\angle C = \angle X$ $\angle D = \angle Y$

iv. Yes. Corresponding sides and angles are equal.
kite ABCD $\cong$ kite ZWXY

Practice Exercise 9

1. A and G, B and H, C and D, E and F.

2. (a) Congruent: RHS: Hypotenuse and corresponding sides are congruent.

   (b) Congruent: AAS: two angles and the side opposite these angles are congruent to the corresponding two angles and side in the other.

   (c) Not congruent: corresponding angles are equal but corresponding sides are not equal.

3. i. (a) Congruent: SAS. Two sides and the included angle of $\Delta\mathbf{A}$ are congruent to the corresponding two sides and the included angle of $\Delta\mathbf{B}$.
$\Delta\mathbf{A} \cong \Delta\mathbf{B}$

   ii. (a) Congruent: SSS. All three corresponding sides are equal.
$\Delta\mathbf{ABC} \cong \Delta\mathbf{XYZ}$

END OF TOPIC 2

TOPIC 3

CIRCLES

TOPIC 3: CIRCLES

Introduction

You have studied circles in Grade 7 and 8 Mathematics. Like other plane figures with geometric properties specific to the figures, the circle, too, has specific properties. It is the most symmetrical of all mathematical curves, and whatever the size, all circles measure 360º.

We see circles being used in real life every day. Some items would include a tire or a wheel, a pizza, a clock, gears, food, dinner plates and more.

Clock

gear

garden

pizza

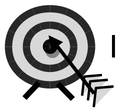
Coys Wheels

You can find circle all over the place in life.

This Topic deals with the different parts and properties of a circle. We will look at how the length of the radius and diameter of a circle are linked to the length of the circumference and the area of the whole circle. We also look at how to find the area of a sector and segments of a circle.

Lesson 10: The Circle and Its Parts

In your Grade 8 lessons, you were introduced to circles and its parts. You also learnt about the circumference and the area of a circle.



In this lesson, you will:

- define a circle
- name other parts of a circle like arc, sector, tangent and secant
- define concentric circles

A circle is an important shape in the field of geometry. Look at the environment around you, whether in the house, in your school, in the village where you are at now. Observe and name some objects that are in the shape of a circle. Are you able to name some?

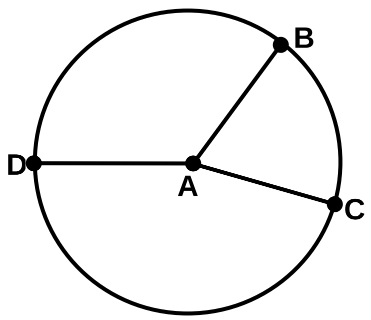
Some real world examples of circles are wheel, a dinner plate, a surface of a coin.

In this lesson we will look at the definition of a circle and its parts. We will identify each part and be able to draw them in the circle as well.

Definition of Circle

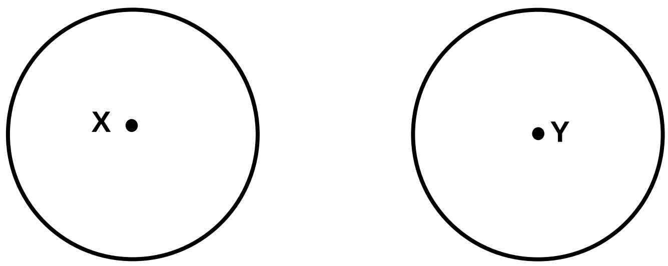
A circle is a shape with all points the same distance from a point called the **centre**. A circle is named by its centre. Thus , the circle in Figure 1 is called circle A since its centre is at point A .

Figure 1



Points B, C and D have the same distance from the centre This means that any point on the circle is **equidistant** from the centre.

The circles below are named as follows:



Circle X Circle Y

The Circle and its Parts

- The **diameter** is the distance across the circle through the centre. This is a line that connects two points on a circle and passes through the centre of the circle.

Figure 2 shows the diameter of a circle.

How many diameters do you think we can draw in a circle?

Are you able to count?

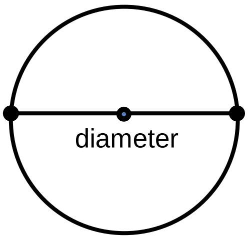
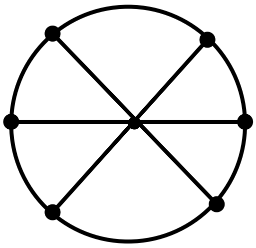


Figure 2

You are right! You can draw as many diameters as you can in a circle. In other words, there can be an infinite number of diameters in a circle.



- The **radius** of a circle is the distance from the centre of a circle to any point on the circle. This is a line that connects the radius and a point on the circle.

Figure 3

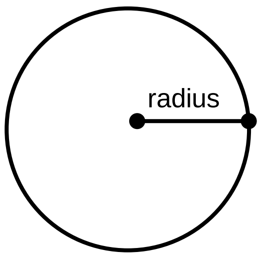
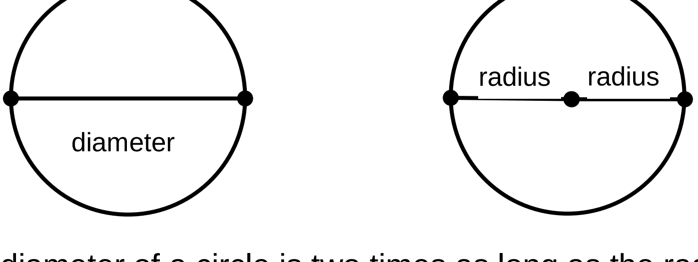


Figure 3 shows the radius of a circle.

Like the diameter, we can also draw an infinite number of radii in a circle.

If you place two radii (plural for radius) end to end in a circle, what happens to the length of the two radii?

Yes you are right! The length of the two radii will have the same length as one diameter.



Thus the diameter of a circle is two times as long as the radius.

In symbols, **D = 2 x r,** where **D** is the diameter and **r** is the radius.

We can also say that the radius is half the length of a diameter.

In symbols, $\mathbf{r = D \div 2}$

- A **chord** is a line segment that joins two points on a curve. In geometry, a chord is often used to describe a line segment joining two endpoints that lie on a circle. The circle in Figure 4 contains **chord XY.**

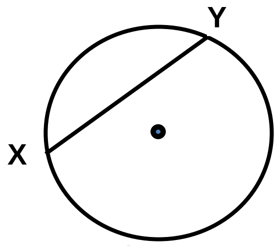
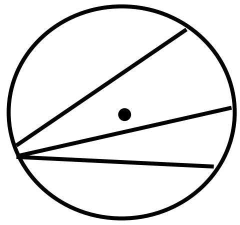
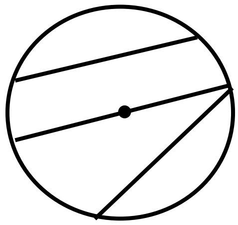


Figure 4



A circle has many different chords.
Some chords pass through the centre and some do not.

A chord that passes through the centre is called a diameter.



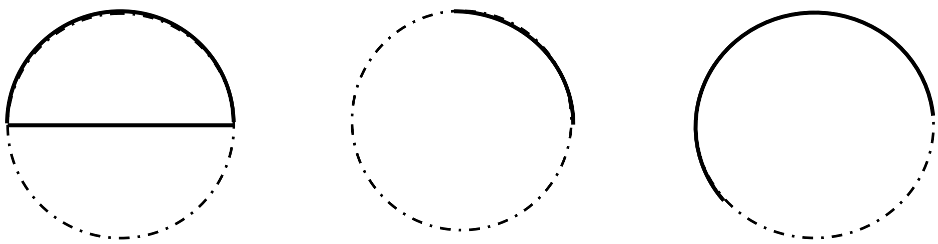
It turns out that the diameter of a circle is the longest chord of the circle since it passes through the centre.

A diameter satisfies the definition of a chord. However, a chord is not necessarily a diameter. This is because every diameter passes through the centre of a circle.

Thus, it can be stated, every diameter is a chord but not every chord is a diameter.

- An **arc** is a part of the circumference of a circle. In Grade 8 you learnt that the circumference is the total distance around the circle. Figure 5 shows the different arcs in a circle.

Figure 5



A **semicircle** is half of the circumference of a circle

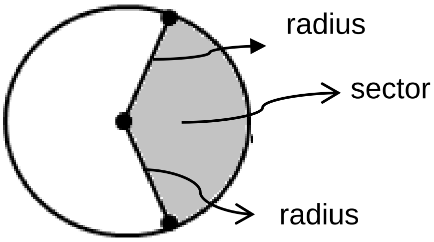
A **minor arc** is smaller than a semicircle.

A **major arc** is bigger than a semicircle.

On the next page are other parts of a circle. These are new words to you so study them carefully.

- A **sector** is a part of the area of a circle enclosed by a radius, an arc and a second radius. Figure 6 (shaded area) shows a sector of a circle.

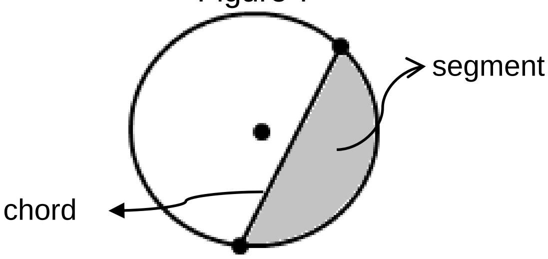
Figure 6



- A **segment** is a part of the area of a circle enclosed by a chord and an arc.

  Figure 7 shows a segment (shaded area) of a circle.

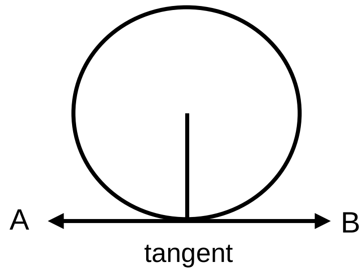
Figure 7



- A **tangent** is a straight line which touches the circle at **only one** point .

  Figure 8 shows the tangent line AB.

Figure 8



- A **secant** is a straight line which cuts a circle at two points. That is an extended chord.

  Figure 9 shows secant line AB.

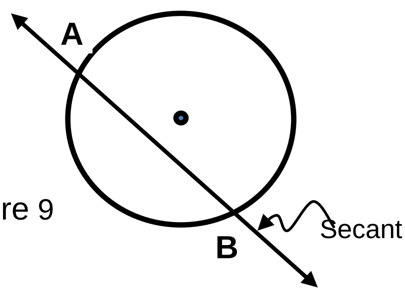


Figure 9

Look at the circles in Figure 10. What is common to all of them?

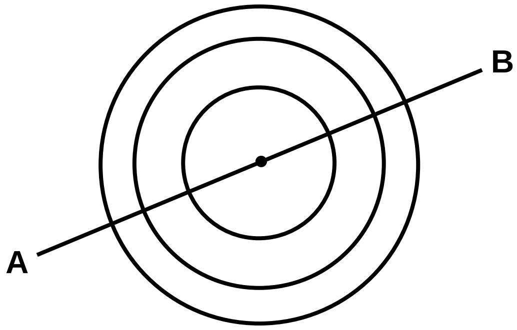


Figure 10

Yes, all these circles have the same centre. They are symmetrical about the line AB and any other diameter of one of the circles. We call these circles **concentric circles**.

Concentric Circles are circles with a common center.

Concentric circles are **circles** of different sizes that all have the same center -- like a bull's eye target.

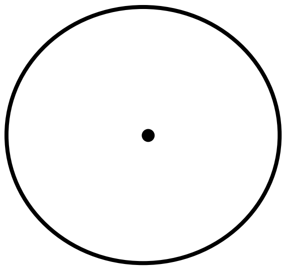
NOW DO PRACTICE EXERCISE 10

Practice Exercise 10

1. Match the name and the definition of the different parts of a circle by drawing arrows between the name and the definition. The first one is done as an example.

a. radius	part of the circumference of a circle
b. secant	part of the area of a circle enclosed by two radii and an arc
c. chord	the distance from the centre to any point on the circle
d. segment	a straight line which cuts a circle at two points
e. sector	part of the area of a circle enclosed by a chord and an arc
f. arc	a straight line which joins any two points on a circle

2.

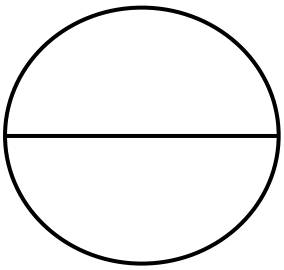


On the circle above, draw the following parts by drawing and labelling them.

(a) tangent MN
(b) secant ED
(c) chord YZ
(d) minor arc AB

3. Given the following measurements find :

(a)



Given: r = 30 cm

Find: D = __________

4. Find r :

(a) D = 13 cm

r = ___________

(b) D = 22 cm

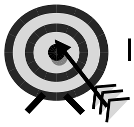
r = ___________

5. What are concentric circles. Draw 4 concentric circles.

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 3.

Lesson 11: Circumference of the Circle

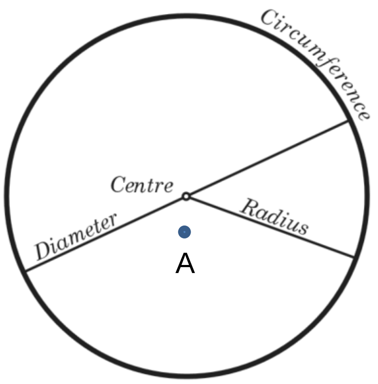
In Grade 8, you learnt about the circumference of a circle and how to use the radius and the diameter to find the distance around a circle.



In this lesson, you will:

- revise your knowledge on how to find the circumference using the formula
- calculate the arc length of a circle

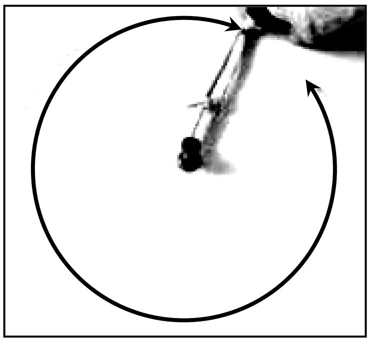
In past lessons, you learned that a circle is a shape with all points the same distance from the centre. The circle below is called circle A since the centre of the circle is at point A.



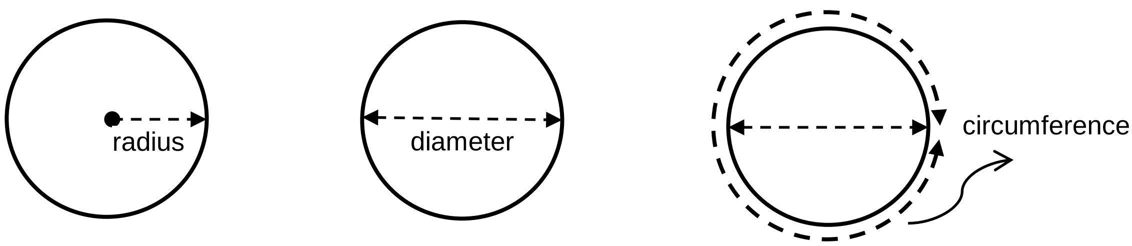
A circle is easy to make:
Draw a curve that is "radius" away from a central point.
And so:
All points are the same distance from the centre.

You can draw it yourself .

Put a pin in a board, put a loop of string around it, and insert a pencil into the loop. Keep the string stretched and draw the circle



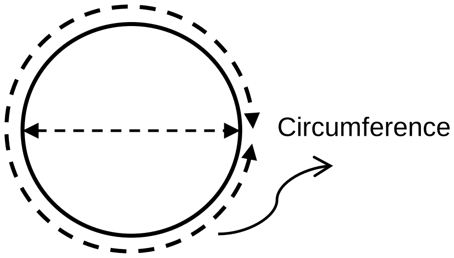
Radius, Diameter and Circumference



The **Radius** is the distance from the centre to the edge.

The **Diameter** starts at one side of the circle, goes through the centre and ends on the other side.

The Circumference of a Circle



The **circumference** is the distance around the edge of the circle.

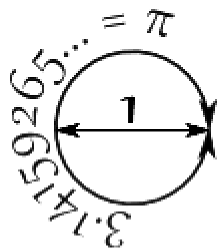
And here is the really cool thing!

When you divide the circumference by the diameter you get 3.141592654... which is the number π **(Pi).**

So when the diameter is 1, the circumference is 3.141592654.

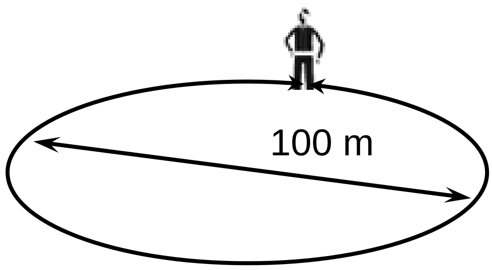
We can say:

$\boxed{\textbf{Circumference} = \pi \times \textbf{Diameter}}$



Example 1

You walk around a circle which has a diameter of 100 m, how far have you walked?



Distance walked = Circumference

= π × diameter

= π x 100 m

= **314 m** (to the nearest m)

Also, note that the Diameter is twice the Radius: Diameter = 2 × Radius

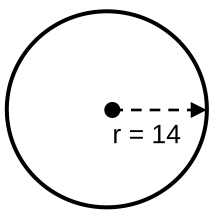
And so this is also true:

Circumference = 2 × π × Radius or C = 2πr

Example 2

A circular swimming pool has a radius of 14 m.

Find the circumference of the pool.



Note: If the radius (**r**), is a multiple of 7, we may use $\frac{22}{7}$ as an approximation value for π.

Solution:

$C = 2\pi r$

$C = 2 \times \frac{22}{7} \times 14$

$C = 88$ m

Example 3

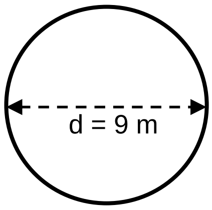
A circular flower-bed has a diameter of 9 m. Find the circumference of the flower-bed.

Solution:

$C = \pi d$

$C = 3.14 \times 9$

$C = 28.26$ m



Sometimes, the circumference of a circle is given and we are asked to find its diameter or radius. In such cases, we can either use $C = 2\pi r$ or $C = \pi d$

To find the **diameter**, we use $C = \pi d$ Make d the subject $C = \pi d$ $C = \frac{\pi d}{\pi}$ $\frac{C}{\pi} = d$	To find the **radius**, we use $C = 2\pi r$ Make r the subject $C = 2\pi r$ $C = \frac{2\pi r}{2\pi}$ $\frac{C}{2\pi} = r$

Example 4

Find the diameter of a circle with a circumference of 44 cm. Use $\pi = \frac{22}{7}$.

Solution: $d = \frac{C}{\pi} = \frac{44}{\frac{22}{7}}$

$= 44\left(\frac{7}{22}\right)$

$= 14$ cm

The diameter is 14 cm.

Example 5

A circle has a circumference of 31.4 cm. What is its radius? Use $\pi = 3.14$.

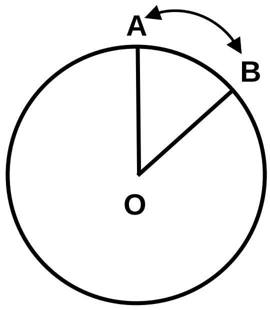
Solution: $r = \dfrac{C}{2\pi}$

$= \dfrac{31.4}{2(3.14)}$

$= 5$ cm

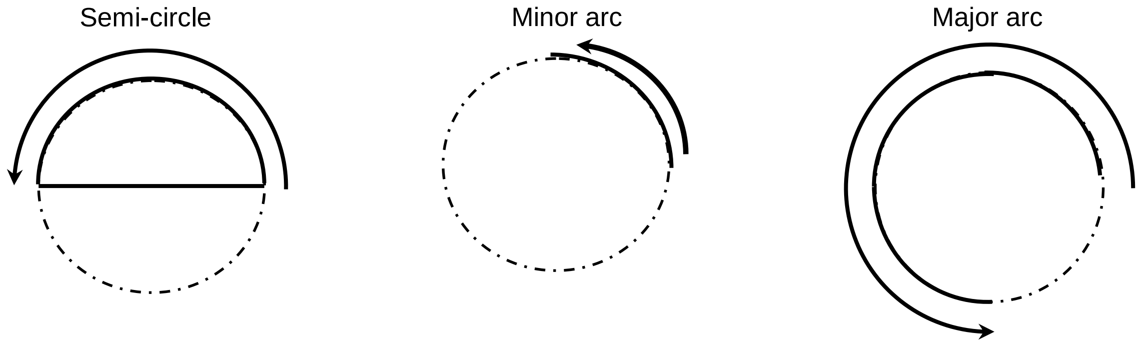
The Length of an Arc

An **arc** is any connected part of the circumference of a circle.



In the diagram above, the part of the circle from A to B forms an arc. It is called arc AB.

In the previous lesson, you learnt about the different arcs.



An arc could be a minor arc, a semicircle or a major arc.

- A semicircle is an arc that is half a circle.
- A minor arc is an arc that is smaller than a semicircle.
- A major arc is an arc that is larger than a semicircle.

Measure of an arc

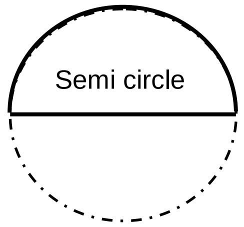
The circumference is the distance around the circle. The measure of the distance all the way around a circle is also the length of the arc in a circle.

So the length of an arc or the arc length for the whole circle is the same as the circumference which is equal to the diameter multiplied by pi.

What if we want to know the length or distance of an arc?

To find the distance of an arc, we divide the total circumference according to the number of divisions we want the circle to be divided into parts.

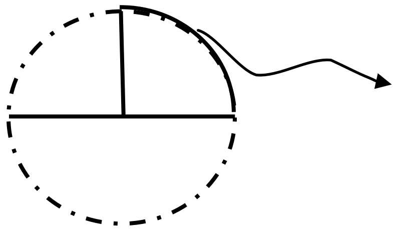
That means the measure of the distance half way around the circle is equal to the diameter times pi divided by 2.



Length of a semicircle = Circumference ÷ 2

Note: In a circle there are two semicircles this is why the circumference is divided by 2.

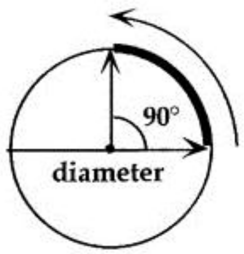
One fourth or a quarter of the way around a circle would be …



Length of an arc = Circumference ÷ 4

You learnt in Grade 8 that all of the way around the circle is 360 degrees and one fourth or one quarter of the way around the circle is 90 degrees. We can write that last one as:

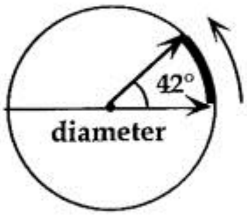
Length of $\frac{1}{4}$ of the way around the circle is = diameter x π x $\frac{90}{360}$.



Great! What's the big deal about that?

The big deal is this. As long as we only want to know stuff like the length of half way around or a quarter of the way around the circle we don't have much problem.

But, what if we want to know the length of a piece of circle that makes an angle of 42 degrees at the centre? Like this:



Example 1

To find the length that we need:

Length = diameter x π x $\frac{42}{360}$

With this, we can find the length of any arc of any circle.

Example 2

Find the length of a piece of circle that makes an angle of 35 degrees at the centre. The radius of the circle is 6. Express the answer to one decimal place.

Since the radius of the circle is 6, the diameter is 2 x 6 = 12.

Solution:

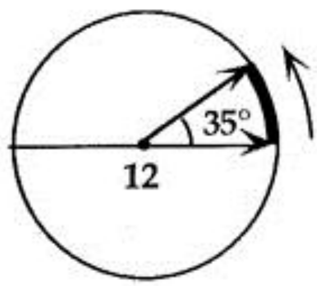
Length = diameter x π x $\frac{35}{360}$

= 12 x π x $\frac{35}{360}$

= $\frac{420\pi}{360}$

= $\frac{7\pi}{6}$

= 3.663 unit



NOW DO PRACTICE EXERCISES 11

Practice Exercise 11

1. What is the circumference of a circle with a radius of 5?

2. A circle's circumference is 102 metres. What is the diameter of the circle?

3. A circle's circumference is 22π metres . What is the radius of this circle?

4. Find the length of a piece of circle that makes an angle of 15 degrees at the centre. The radius of the circle is 6.

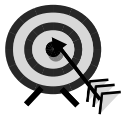
5. Find the length of a piece of circle that makes an angle of 110 degrees at the centre. The radius of the circle is 10.

6. Find the length of a piece of circle that makes an angle of 300 degrees at the centre. The diameter of the circle is 14.

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 3

Lesson 12: Area of the Circle

In Grade 8, you learnt about the areas of a circle using the formula $A = \pi r^2$.



In this lesson you will:

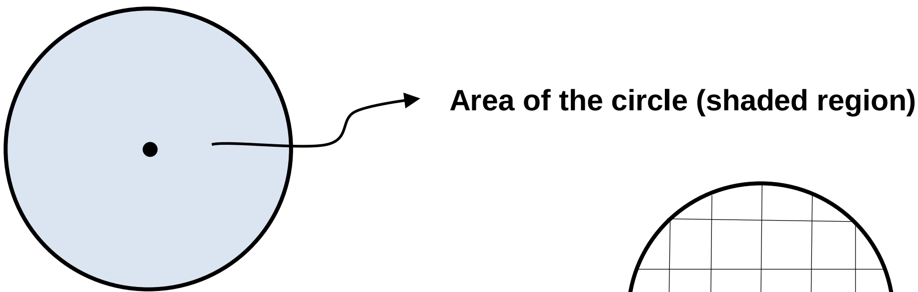
- revise how to calculate the area of a circle
- calculate the area of a semicircle and a quarter circle using the area of a circle
- calculate the area of a sector.

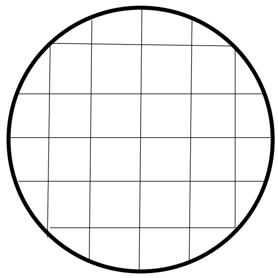
The following are highlights from the previous lesson:

- A **circle** is a closed curve formed by a set of points on a plane that are the same distance from its centre.
- The distance around a circle is called its **circumference**. The distance across a circle through its centre is called its **diameter**.
- We use the Greek letter **π** (pronounced **Pi**) to represent the ratio of the circumference of a circle to the diameter.
- In the last lesson, we learned that the formula for circumference of a circle is: **C = π d** or **C = 2 π r**. For simplicity, we use π = 3.14 or $\frac{22}{7}$.
- We know from the last lesson that the diameter of a circle is twice as long as the **radius.** This relationship is expressed in the following formula: d = 2 x r..

The area of a circle

The area of a circle is the region enclosed by the circle.





The area of a circle is the number of square units inside that circle. If each square in the circle to the left has an area of 1 cm^2, you could count the total number of squares to get the area of this circle.

Thus, if there were a total of approximately 28 squares, the area of this circle would be 28 cm^2.

However, it is easier to use one of the following formulas:

The area of a circle is given by the formula:

$$A = \pi r^2$$

where *A* is the area and *r* is the radius.

Since the formula is only given in terms of radius, remember to change from diameter to radius when necessary. The radius is equal to half the diameter.

Area of a circle given the diameter or radius

Example 1

Find the area of the circle with a diameter of 10 inches.

Solution:

Step 1: Write down the formula: $A = \pi r^2$

Step 2: Change diameter to radius: $r = \frac{1}{2}d = \frac{1}{2} \times 10 = 5$

Step 3: Plug in the value: $A = \pi \times 5^2 = 25\pi$

Answer: The area of the circle is $25\pi \approx 78.55$ square inches.

Example 2

Find the area of the circle with a radius of 10 inches.

Solution:

Step 1: Write down the formula: $A = \pi r^2$

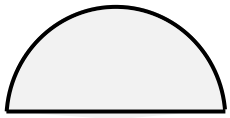
Step 2: Plug in the value: $A = \pi \times 10^2 = 100\pi$

Answer: The area of the circle is $100\pi \approx 314.2$ square inches.

The area of a semicircle and a quarter circle

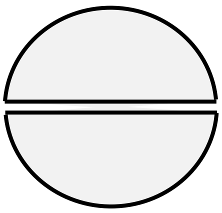
In the previous lesson you learnt that:

A semicircle is half of a circle. It is made up of the diameter and half the circumference of a circle.

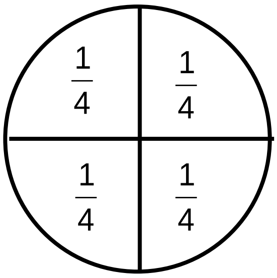


The area of a semicircle is HALF the area of a circle.

$$A = \frac{\pi r^2}{2}$$

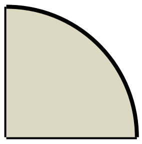


A quarter-circle is one fourth of a circle. It is made up of the 2 radii and an arc.



The area of a quarter-circle is the area of a circle divided by 4.

$$\mathbf{A = \frac{\pi r^2}{4} \text{ or } \frac{1}{4}\pi r^2}$$



Example 1

Find the area of a semicircle with a radius of 7 cm. Use π = 3.14

$$\text{Area of semicircle} = \frac{\pi r^2}{2}$$

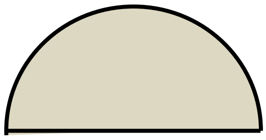
$$= \frac{\frac{22}{7} \times 7^2}{2}$$

$$= \frac{\frac{22}{7} \times 49}{2}$$

$$= \frac{22 \times 7}{2}$$

$$= 11 \times 7$$

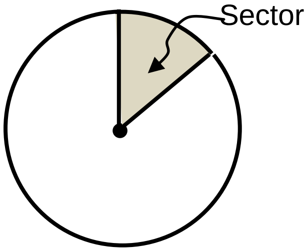
$$= 77\text{cm}$$



The area of the semicircle is 77 cm^2.

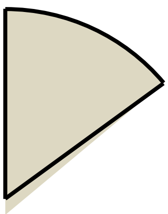
The area of a sector

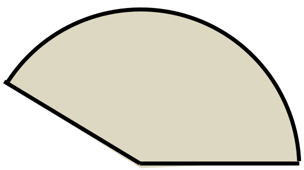
Let us begin by looking at what a sector is.

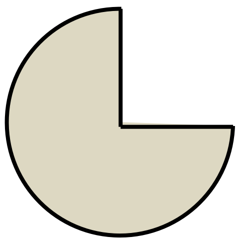


A sector is a part or a fraction of a circle bounded by two radii and an arc.

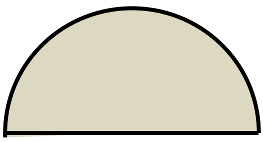
A sector of a circle is a pie shaped portion of a circle.

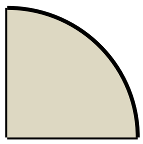






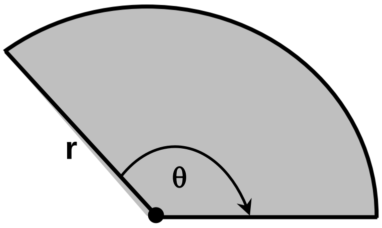
So, semicircles and quarter circles are also sectors.





To find the area of a sector of a circle, we use the formula:

$$\boxed{\mathbf{A = \pi r^2 \times \frac{\theta}{360}}}$$

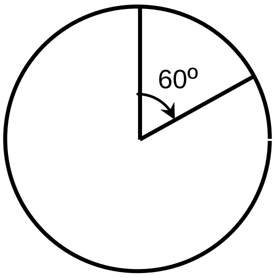


Where, **r** is the radius and θ is the central or sector angle.

The central angle is the opening formed by the two radii as shown in the diagram.

Example 1

Find the area of a sector with a central angle of 60 degrees and a radius of 3 cm. Use π = 3.14.



To work out the area of the sector, we do the following steps.

a) Find what fraction of the whole circle 60° is. The complete angle at the centre of a circle is 360°.

b) Use that fraction of the whole circle to calculate the area of the shaded circle.

Solution:

a) Find what fraction of the whole circle is 60^0

$$\text{Sector fraction} = \frac{\theta}{360} = \frac{60}{360} = \frac{6}{36} = \frac{1}{6}$$

b) Now, we can calculate the area of the sector using the formula.

$$\text{Area of the sector} = \pi r^2 \text{ x } \frac{\theta}{360}$$

$$= 3.14 \text{ x } 3^2 \text{ x } \frac{1}{6}$$

$$= 3.14 \text{ x } 9 \text{ x } \frac{1}{6}$$

$$= \frac{28.26}{6}$$

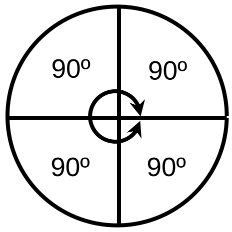
$$= 4.71 \text{ cm}^2$$

Let us look at the sector angle of a semicircle and a quarter-circle

A circle has 360º angle.

A semicircle has a 180º sector angle.

A quarter-circle has a 90º sector angle.



To find the area: Semi circle Quarter circle

Semi circle

$$\mathbf{A = \pi r^2 \times \frac{\theta}{360}}$$

$$\mathbf{A = \pi r^2 \times \frac{180}{360}}$$

$$\mathbf{A = \pi r^2 \times \frac{1}{2} \quad or \quad \frac{\pi r^2}{2}}$$

Quarter circle

$$\mathbf{A = \pi r^2 \times \frac{\theta}{360}}$$

$$\mathbf{A = \pi r^2 \times \frac{90}{360}}$$

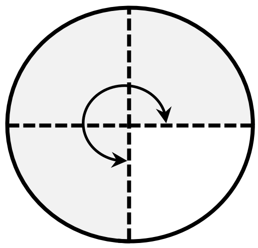
$$\mathbf{A = \pi r^2 \times \frac{1}{4} \quad or \quad \frac{\pi r^2}{4}}$$

Example 2

Find the area of the shaded sector shown in the circle below. Use π = 3.14.

Solution: Since there are 3 quarters shaded in the circle the sector angle is 270^0.

Area of the sector = $\pi r^2 \text{ x } \frac{\theta}{360}$



$= 3.14 \text{ x } 2^2 \text{ x } \frac{270}{360}$

$= 3.14 \text{ x } 4 \text{ x } \frac{3}{4}$

$= 3.14 \text{ x } 4 \text{ x } 0.75$

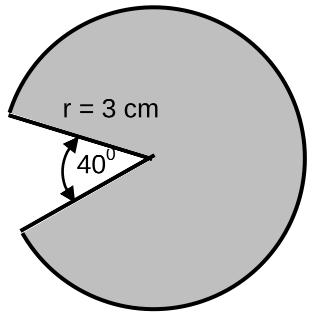
$= 9.4 \text{ cm}^2$

In the next example, the sector angle of the shaded sector is not given. We are instead given the angle of the unshaded sector.

Example 3

Find the area of the shaded sector of the circle below. Use π = 3.14.

Solution:



Find the sector angle required.

Sector angle required = 360º – 40º = 320º

So 320º is the angle of the shaded sector.

So the area of the shaded sector is

$$A = \pi r^2 \times \frac{\theta}{360}$$

$$A = 3.14 \times 3^2 \times \frac{320}{360}$$

$$A = 3.14 \times 9 \times \frac{8}{9}$$

$$A = 3.14 \times 9 \times 0.88$$

$$A = 25.1 \ \text{cm}^2$$

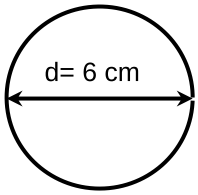
Sometimes you are asked to express your answer correct to a number of decimal places. So be careful to give what is being asked.

NOW DO PRACTICE EXERCISES 12

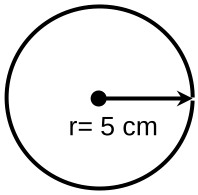
Practice Exercise 12

1. Find the area of each of the following circles. Use $\pi = \frac{22}{7}$.

(a)

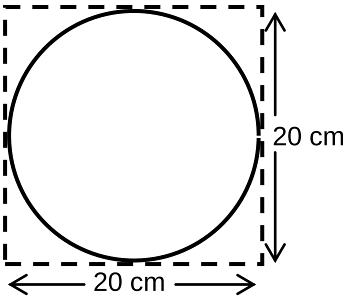


(b)



2. This circular portion of land is a flower garden. Use $\pi = 3.14$.

Write your answer correct to one decimal place.



a) What is the diameter? Answer: __________

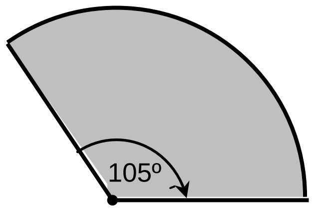
b) What is the radius? Answer: __________

c) What is the area of the flower garden?

Answer: __________

3. Calculate the area of the sector shown below. It has a radius of 2 cm.

Use π = 3.14. Give your answer correct to one decimal place.



Answer:____________

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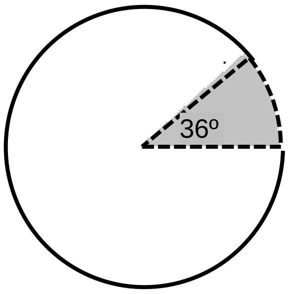
4. Work out the area of the circle below. It has a radius of 7 cm. Use $\pi = \frac{22}{7}$.



(a) Calculate the area of the circle.

Answer: ____________

(b) Calculate the area of the shaded sector.

Answer: ____________

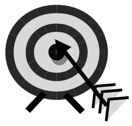
(c) Calculate the area of the unshaded sector.

Answer: ____________

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 3

Lesson 13: Segment of the Circle

In the previous lesson, you have learnt how to find the area of the sector by using the formula for the area of the circle and also the central angle.

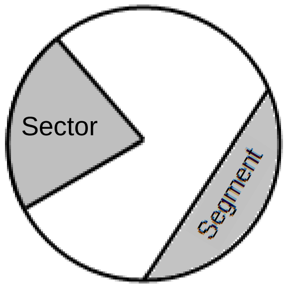


In this lesson you will:

- revise concepts on how to calculate the area of a sector
- calculate the area of a segment of a circle.

There are two main **"slices"** of a circle:

- The "pizza" slice is called a **Sector**.
- And the slice made by a chord is called a **Segment.**



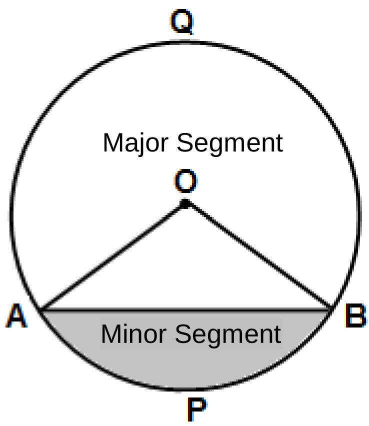
Segment of a Circle

The segment of a circle is a part of the interior of a circle bounded by an intercepted arc and a chord. It can also be defined as a region enclosed by an arc of a circle and the chord connecting the endpoints of the arc.

A line connecting two points on a circle is a **chord** of the circle. It divides the circle into two parts, each called a segment. A segment of a circle lies between the **chord** and **arc** of the circle.

- The unshaded region is the **major** segment.
- The shaded is the **minor** segment.

Note: The radius and centre of the circle are not part of the segment of the circle.



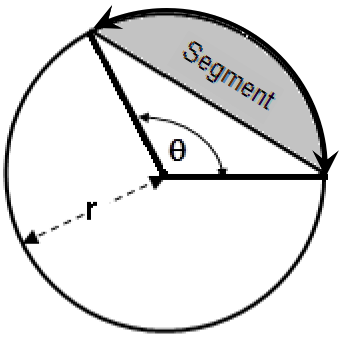
In the above diagram, the shaded part is a portion of the circle bounded by the arc APB and the chord AB. The arc AQP is the remaining portion of the circle (or deemed to be a circle) with centre at O. The shaded area is called the minor segment of the circle.

Area of Segment (angle in degrees)

The segment of a circle is a region bounded by the arc of the circle and a chord.

As explained earlier, a segment of a circle is a part of the area of the circle enclosed by part of its circumference and a chord. Look at the diagram shown below.

The shaded area is the segment of the circle.



Finding the Area of a Segment of a Circle

Dealing with the area of a segment is very similar to working with the area of a sector.

Area of a Minor Segment

Step 1 Work out the area of the minor sector
Step 2 Work out the area of the triangle formed by the chord and the two radii
Step 3 Subtract the area of the triangle **from** the area of the sector

In Formula:

Area (minor segment) = Area (sector) – Area (triangle)

$$\textbf{Area (segment)} = \pi r^2 \times \frac{\theta}{360} - \frac{1}{2}bh$$

Area of the major segment

Step 1 Work out the area of the major sector
Step 2 Work out the area of the triangle
Step 3 Add the area of the sector and the triangle

In formula:

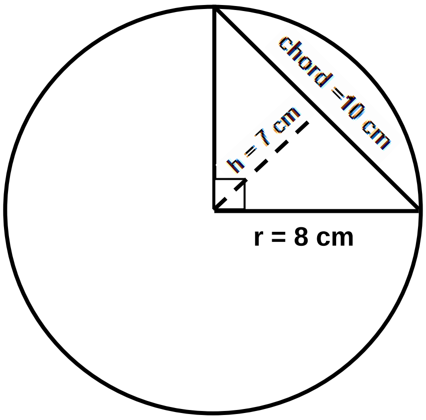
Area (major segment) = Area (sector) + Area(triangle)

$$\textbf{Area(segment)} = \pi r^2 \times \frac{\theta}{360} + \frac{1}{2}bh$$

Let's look at some example problems.

Example 1

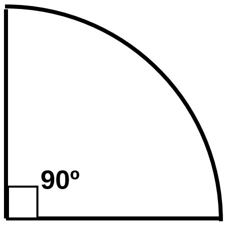
Find the area of the minor segment of a circle with a central angle of 90 degrees and a radius of 8, a perpendicular height of 7 and a chord of 10. Express answer to nearest whole number..



Solution

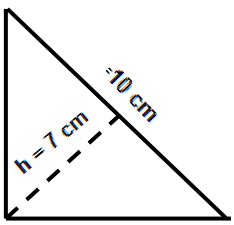
Step 1: Start by finding the area of the minor sector:

$$A = \pi r^2 \times \frac{\theta}{360}$$
$$A = 3.14 \times 8^2 \times \frac{90}{360}$$
$$A = 3.14 \times 64 \times \frac{1}{4}$$
$$A = 50.24$$



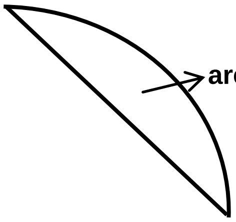
Step 2: Now, find the area of the triangle.

$$A = \frac{1}{2}bh$$
$$A = \frac{1}{2} \times 10 \times 7$$
$$A = 35$$



Step 3: Find the area of the segment.

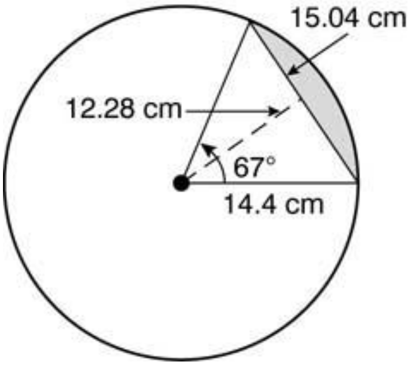
$$A_{segment} = A_{sector} - A_{triangle}$$
$$A = 50.24 - 35$$
$$A = 15.24$$
$$A = 15$$



Therefore the area of the minor segment is 15.

Example 2

Find the area of the shaded segment of the given circle. Write your answer correct to one decimal place.



Solution:

Step 1 Area of the sector

$$A = \pi r^2 \times \frac{\theta}{360}$$
$$A = 3.14 \times 14.4^2 \times \frac{67}{360}$$
$$A = 3.14 \times 207.36 \times 0.19$$
$$A = 123.7\text{cm}^2$$

Step 2 Area of the triangle

$$A = \frac{1}{2}bh$$
$$A = \frac{1}{2} \times 15.04 \times 12.28$$
$$A = \frac{184.69}{2}$$
$$A = 92.3 \text{ cm}^2$$

Step 3 Area of the segment

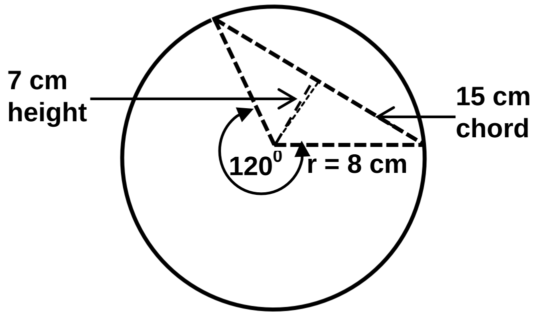
$$A_{\text{segment}} = A_{\text{sector}} - A_{\text{triangle}}$$
$$A = 123.7 - 92.4$$
$$A = 31.3 \text{ cm}^2$$

Therefore the area of the minor segment is 31.3 cm^2.

Now we will see some examples on working out the area of the major sector.

Example 3

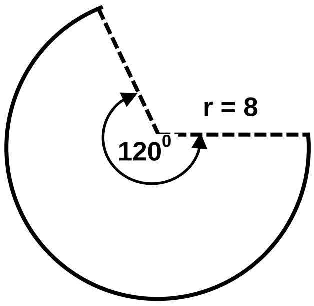
Find the area of the major segment in the diagram below.



Solution:

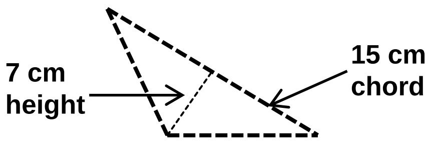
Step 1: Find the area of the major sector

$$A = \pi r^2 \times \frac{\theta}{360}$$
$$A = 3.14 \times 8^2 \times \frac{120}{360}$$
$$A = 3.14 \times 64 \times 0.33$$
$$A = 66.3\,\text{cm}^2$$



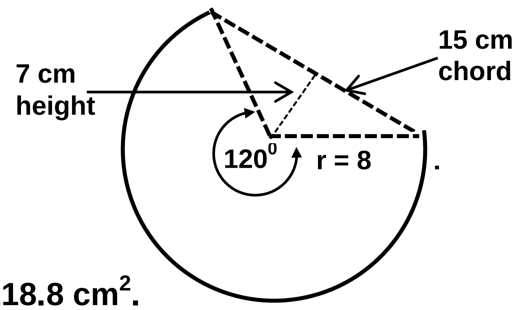
Step 2: Find the area of the triangle

$$A = \frac{1}{2}bh$$
$$A = \frac{1}{2} \times 15 \times 7$$
$$A = \frac{105}{2}$$
$$A = 52.5\,\text{cm}^2$$



Step 3: Find the area of the major segment

$$A_{\text{segment}} = A_{\text{sector}} + A_{\text{triangle}}$$
$$A = 66.3 + 52.5$$
$$A = 118.8\,\text{cm}^2$$

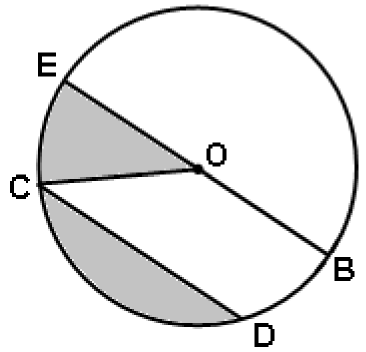


Therefore the area of the minor segment is 118.8 cm^2.

NOW DO PRACTICE EXERCISE 13

Practice Exercise 13

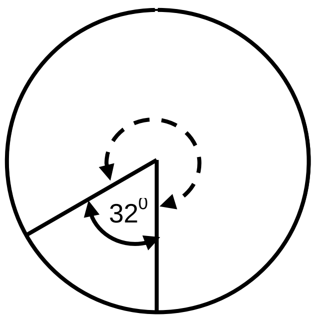
1. Match the parts of this circle with their correct names. (Point **O** is the centre of the circle.) Use an arrow to connect the correct pair.



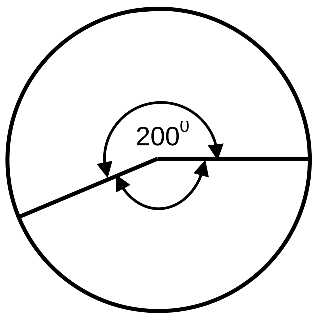
Column **A**	Column **B**
a) EB	1. radius
b) OC	2. diameter
c) ECD	3. sector
d) EOC (shaded)	4. segment
e) CD (shaded)	5. arc

2. Find the sector angle of the circle using the given angle.

(a) Minor central angle = 32^0 Major central angle = ___________

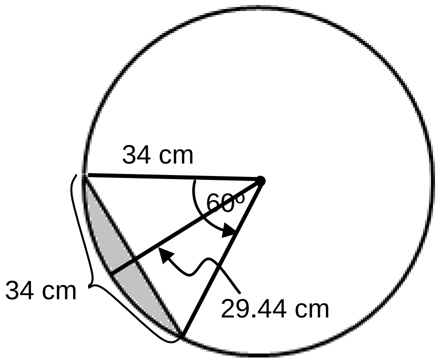


(b) Major central angle = 200^0 Minor central angle = ___________



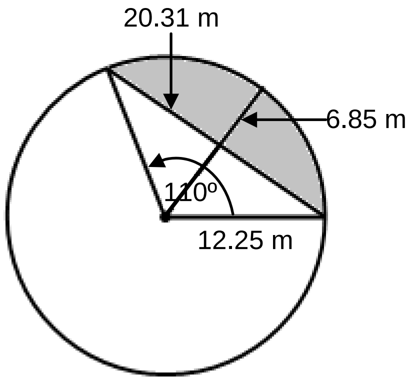
\

3. Find the area of the shaded segment of the given circle. Express answer correct to one decimal place.



Answer: ___________

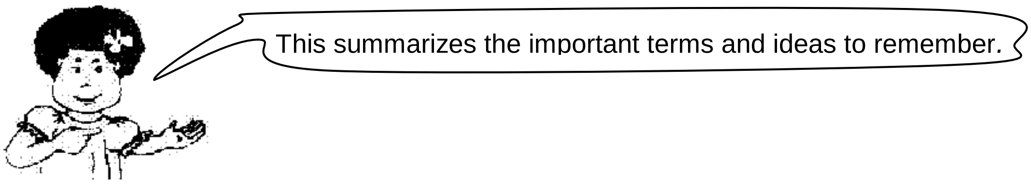
4. Find the area of the shaded segment of the given circle. Express answer correct to one decimal place.



Answer: ___________

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 3

TOPIC 3: SUMMARY



- **The Circle** is a shape with all points of which are the same distances from the centre.
- **The Diameter** is a straight line segment joining two points on the circle passing through the centre.
- **The Radius** is a straight line segment from the centre to any point on the circle.
- **A Chord** is a straight line segment that joins two points on a circle.
- **An Arc** is a part or portion of a circle.
- **A Semicircle** is an arc that is equal to half a circle.
- **A quarter-circle** is one-fourth of a circle.
- **A Minor arc** is an arc less than a semicircle.
- **A Major arc** is an arc bigger than a semicircle.
- **The Circumference** is the distance around the edge of a circle. It is found by using the formula: $\boxed{C = 2\pi r \text{ or } C = \pi D}$
- The **area of a circle** is the region enclosed by the circle. It is found by the formula;

$$\boxed{A = \pi r^2} \quad \text{or} \quad \boxed{A = \frac{\pi D^2}{4}}$$

- **The Sector of a circle** is the portion of the interior of a circle enclosed by two radii and an arc.
- The **segment of a circle** is the region bounded by a chord and the arc subtended by the chord.
- The **area of a sector of a circle** is found by using the formula:

$$\boxed{A = \pi r^2 \times \frac{\theta}{360}}$$

- The area of a segment of a circle is found by using the formula:

$$\boxed{A_{Segment} = \pi r^2 \times \frac{\theta}{360} - \frac{1}{2}bh} \quad \textbf{for minor segment}$$

$$\boxed{A_{Segment} = \pi r^2 \times \frac{\theta}{360} + \frac{1}{2}bh} \quad \textbf{for major segment}$$

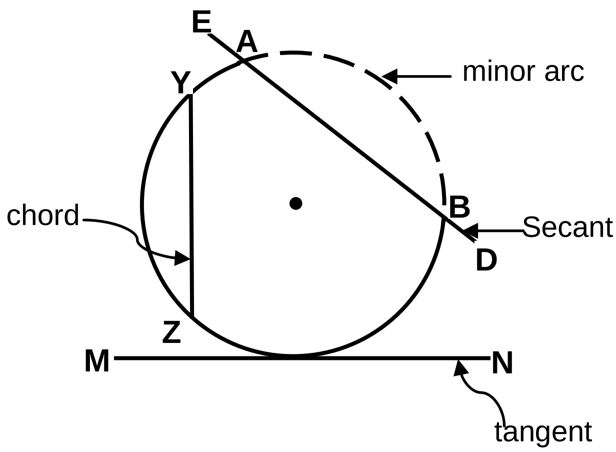
REVISE LESSON 10 TO 13. THEN DO TOPIC TEST 3 ASSIGNMENT BOOK 6

ANSWER TO PRACTICE EXERCISES 10 - 14

Practice Exercise 10

1. radius	part of the circumference of a circle
2. secant	part of the area of a circle enclosed by two radii and an arc
3. chord	the distance from the centre to any point on the circle
4. segment	a straight line which cuts a circle at two points
5. sector	part of the area of a circle enclosed by a chord and an arc
6. arc	a straight line which joins any two points on a circle

2.



3. D = 2 x r

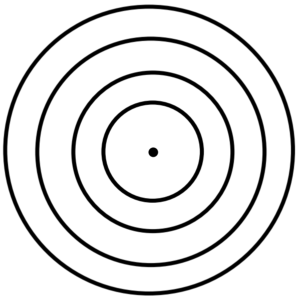
D = 30 x 2

D = 60 cm

4. (a) r = 6.5 cm

(b) r = 11 cm

5. Concentric circles are circles that have the same centre.



Practice Exercise 11

(1) 31.4 (2) 32.5 m (3) 11 m (4) 1.6 (5) 19.2 (6) 36.6

Practice Exercise 12

1. (a) A = 28.3 (b) A = 78.6
2. (a) diameter = 20 cm (b) radius = 10 cm (c) A = 314 cm^2
3. A = 3.7 cm^2
4. (a) A= 154 cm^2 (b) 15.4 cm^2 (c) 138.6 cm^2

Practice Exercise 13

1.

Column **A**	Column **B**
a) EB	1. radius
b) OC	2. diameter
c) ECD	3. sector
d) EOC (shaded)	4. segment
e) CD (shaded)	5. arc

2. (a) 328^0 (b) 160^0

3. Area of the segment = Area of the sector – area of the triangle
= 604.97 – 500.48
= 104.5 cm^2

4. Area of the segment = Area of the sector – area of the triangle
= 143.97 – 55.
= 88,97 cm^2 or 89 cm^2

END OF TOPIC 3

TOPIC 4

CONSTRUCTION

TOPIC 4: CONSTRUCTION

Introduction

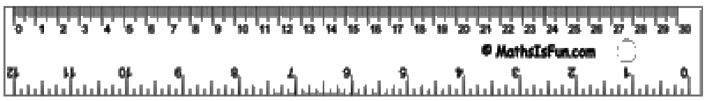
Making accurate drawings is not a skill confined to artists. Draftsmen, architects, engineers, and designers need to be able to construct geometrical figures.

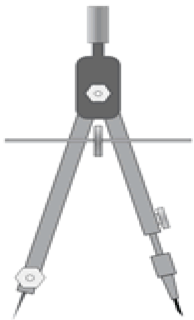
Trying to learn Geometry without using drawing and geometric construction is like trying to learn Chemistry or Biology without using laboratories

Basic knowledge and skills on drawing and geometric constructions help students to discover and explore geometric relationships and interpret geometric concepts and theorems.

Geometric figures are nearly always helpful for the analysis and solutions of real world problems and for learning new mathematical ideas.

Construction is drawing of various geometric shapes like lines, using only compass and straightedge or ruler.

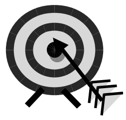




In this Topic, we will look at the different methods of drawing solid objects and how to carry out some common geometric constructions using only a pair of compasses and a ruler.

Lesson 14: Drawing Solid Objects

In your previous lessons in Grade 7 and 8, you learnt about nets to draw a solid figure.



In this lesson you will:

- identify the different methods of drawing solid objects
- use isometric drawing or oblique methods to draw solid objects
- draw the different perspective views of a solid.

Introduction to 3-D Drawing and Geometry

Wherever we look, we see three-dimensional shapes. Buildings, furniture, plants, even people themselves: all are solid objects. Whenever we look at the world around us, we see it in three dimensions: length, width and height. Drawings that are created to represent the idea of these three dimensions are often called "3-D drawings."

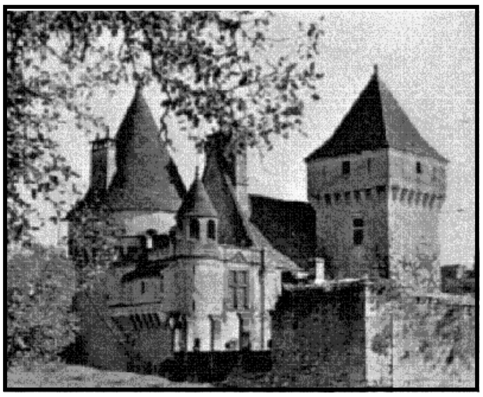
The Geometry of 3-D Drawings

When we draw any object, we have the choice of drawing it "flat" (two-dimensionally) or as a "solid" (three-dimensionally). A floor plan is an example of a two-dimensional representation of a house. Architects often draw 3-D drawings of houses, so their clients can more clearly understand what the house will look like when it is built.

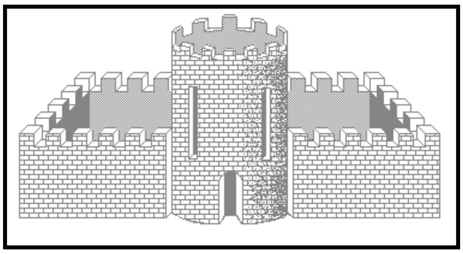
In this lesson, you will be studying three-dimensional geometric objects such as cubes, cylinders and pyramids, and learning how to draw them so that they appear to be 3-D. Below are some examples of the 3-D solids you will be studying:

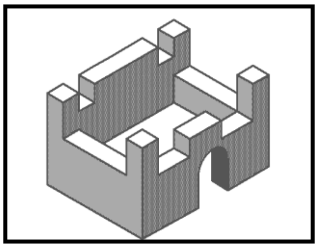
Once you know how to draw these solids, you can combine them to draw all sorts of three-dimensional objects such as furniture, houses, and even castles!

The beautiful photograph below from Castles on the web archive is of a castle called Laussel, at Marquay, in the Perigord region of France.

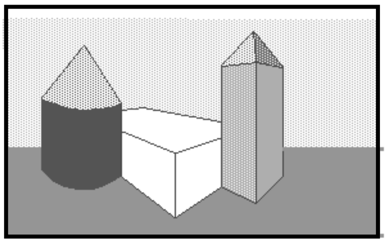


This and many other castles are made up of geometric solids. Can you find prisms, pyramids, cones and cylinders in the photograph? You can draw castles using a combination of these geometric solids. An example of a simple castle, drawn in oblique using Adobe SuperPaint computer software, is shown below:





The castle below was drawn in isometric, using the isometric grid in a computer software.



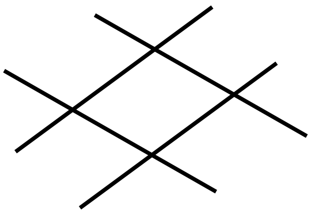
Here's another example of a castle. This one was constructed in perspective, using the Google Sketchup

3-D Drawing Methods

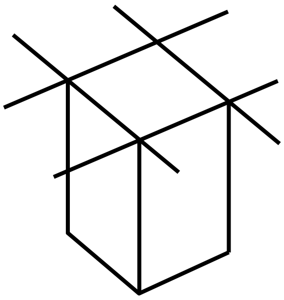
Drawing Method 1: Using Lines

Prisms and pyramids can be drawn using parallel lines. You have studied what parallel lines are in Grade 8.

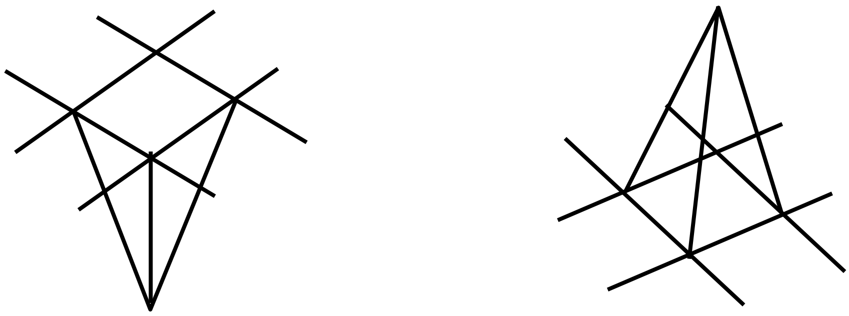
Step 1 Draw two pairs of parallel lines that cross like this.



Step 2 Draw three lines of equal length straight down from the corners. Join the ends to get a prism as shown.

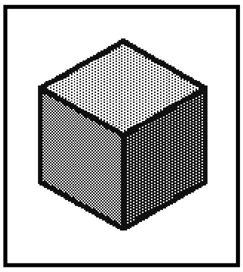


Step 3 Choose a point in the middle, below or above the figure. Draw lines from three corners to this point to get a pyramid as shown.



Drawing Method 2: Isometric and Oblique Drawing

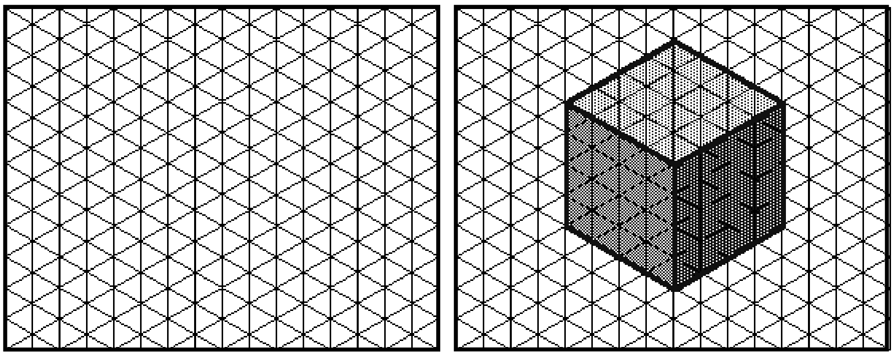
There are many different types of 3-D drawings. One fairly simple way to get started drawing in 3-D is to try an *isometric* drawing. An isometric drawing of a cube looks like this:



An easy way to draw objects in isometric is to use an isometric grid, as shown below.

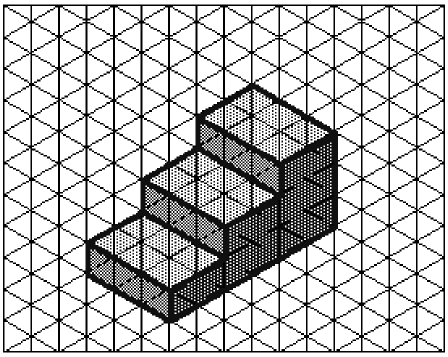
On an isometric grid, there are 3 types of lines: vertical lines, 30° lines to the right, and 30° lines to the left. The drawing below on the left shows an isometric grid; the drawing on the right shows an isometric grid with a cube drawn on it. In all of these drawings, the faces are shaded to make it look more "solid." This kind of shading is not necessary, but you may do it if you like.

Isometric Drawing

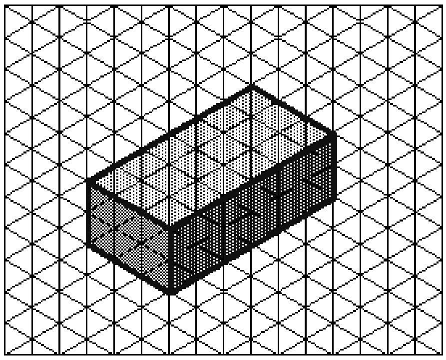


You can use an isometric grid to draw many different shapes. Here are some examples of **different** geometric shapes drawn on isometric grids. You may also print out a large isometric grid and use it to create your own isometric drawings.

Examples of different geometric shapes drawn on isometric grids:

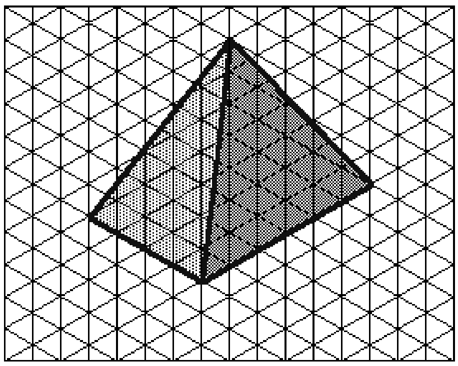


3 stairs

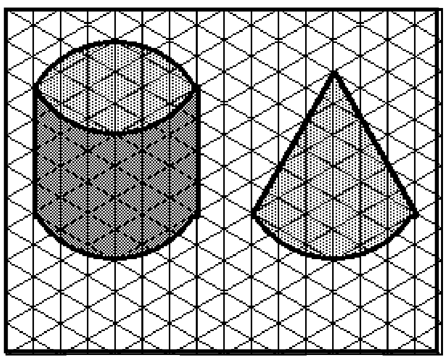


a brick

Sometimes in order to draw a particular shape you need to draw lines that don't fall right on the grid, for example when drawing a pyramid with a rectangular base, as shown below.



To draw a cone or cylinder, you will need to use curved portions, arcs of circles, as shown below:



Cylinder **Cone**

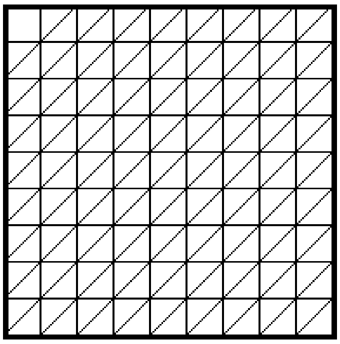
When you use the grid, place a piece of unlined blank paper (or tracing paper) over it and sketch with a pencil on the tracing paper, following the lines of the grid. If you draw on blank paper or tracing paper, you can then remove the grid from beneath your drawing and the drawing will look much nicer than if you draw directly on the grid. Begin with simple shapes and then combine shapes to form more complex drawings.

You can sketch with pencil and paper using the grid, or use computer drawing software to create isometric drawings. You can also construct an isometric using the Google Sketch Up software.

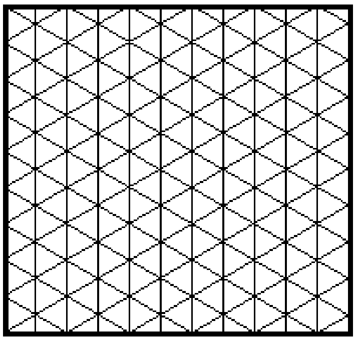
Oblique Drawing

Another type of drawing is an oblique drawing. Compare the two oblique grids below.

What types of lines are the same in each grid? What types of lines are different?



Oblique Grid



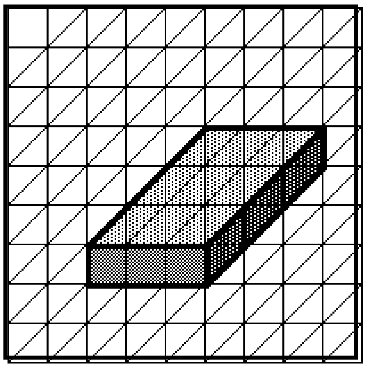
Isometric Grid

If you answered as follows, you are correct:

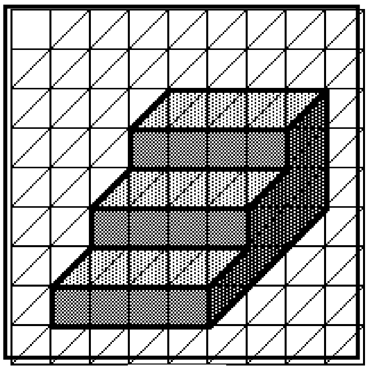
- Both oblique and isometric drawings have a set of vertical lines.
- Oblique drawings have a set of 45° lines and a set of horizontal lines, while isometric drawings have two sets of 30° lines.
- Isometric drawings have no horizontal lines.

You can use an oblique grid to draw many different shapes. Here are some examples of simple shapes drawn using an oblique grid, and a large oblique grid to print out and use to create your own isometric drawings.

Oblique Examples



a box



3 steps

When you use the grid, place a piece of unlined blank paper (or tracing paper) over it and use a pencil to sketch on the tracing paper, following the lines of the grid. If you draw on blank paper or tracing paper, you can remove the grid from beneath your drawing and the drawing will look much nicer than if you draw directly on the grid.

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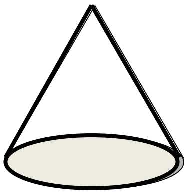
- **Isometric drawings show a corner view.**
- **Oblique drawing shows a front view.**

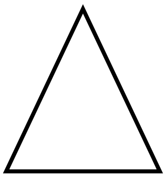
3-D Views of Solid Figures

Perspective view of a cone

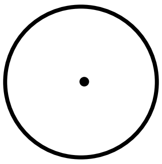


Different angle views of a cone

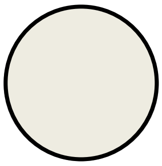
Side view



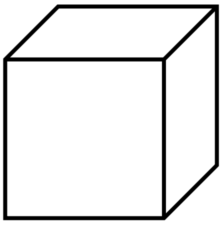
Top view



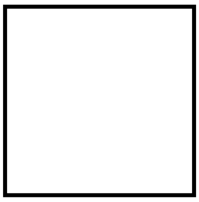
Bottom view



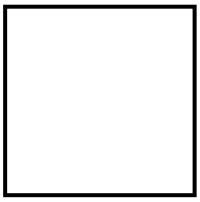
Different views of a Cube



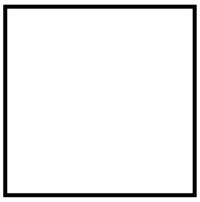
Front view



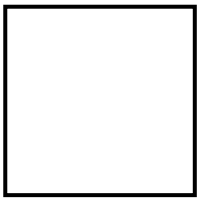
Left view



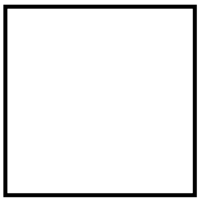
Right view



Back view

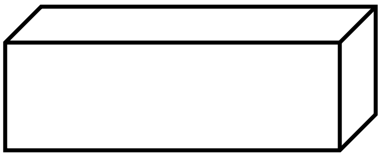


Top view



A cube has all six faces the same which is a square.

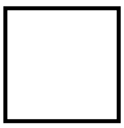
Different views of a rectangular prism



Front/Back

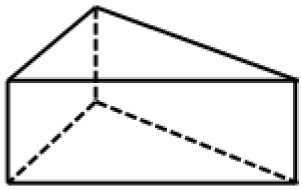
Top/Bottom

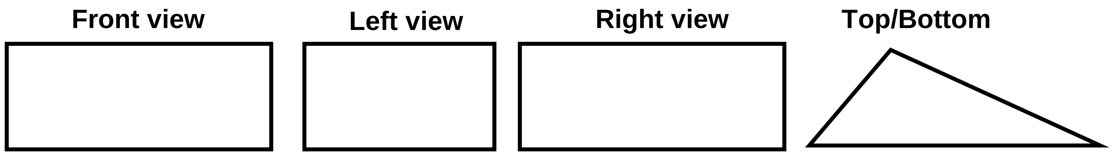
Right/Left



A rectangular prism has a square and rectangle as faces.

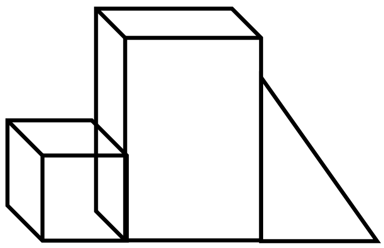
Different views of a triangular prism.

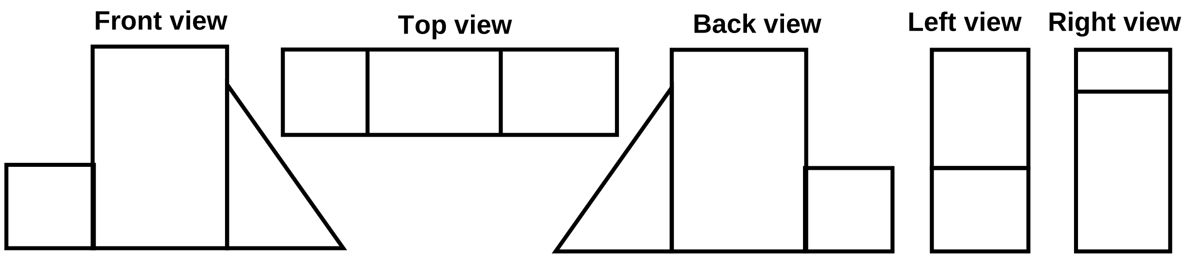




Let us look at composite figure and identify its different views.

Example



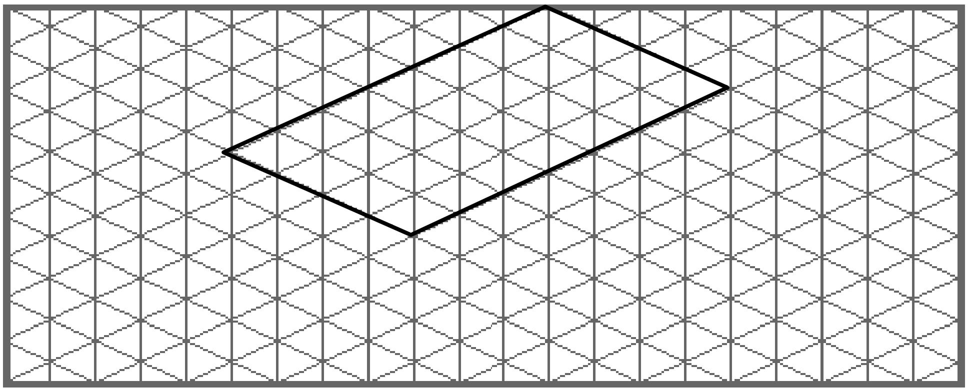


NOW DO PRACTICE EXERCISES 14

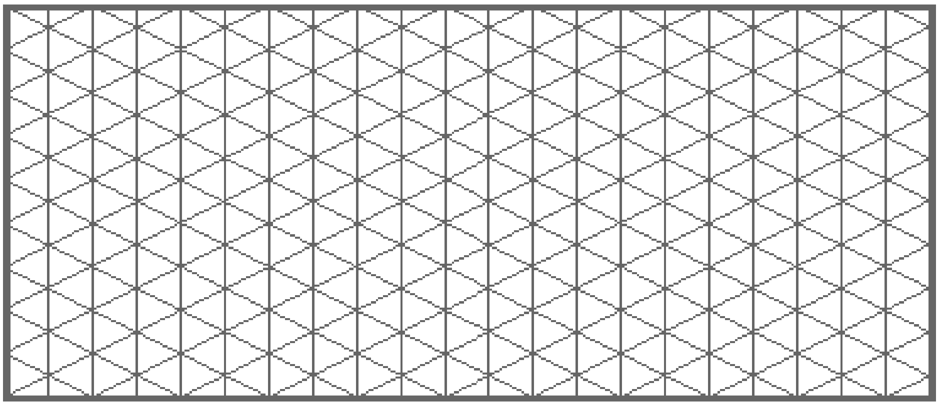
Practice Exercise 14

1. Sketch a rectangular solid 7 units long, 4 units wide and 3 units high using the isometric grid paper below. The first step is done for you.

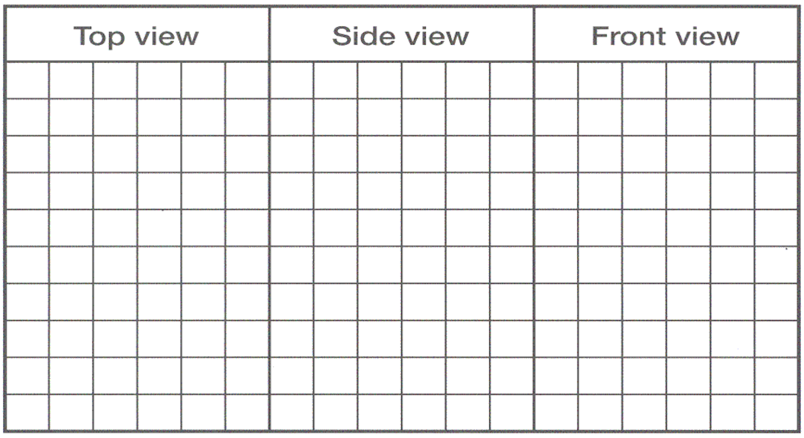
Step 1 Draw the top of the solid 4 by 7 units.



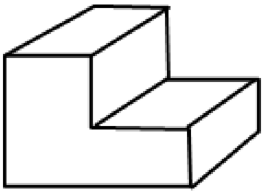
2. Sketch a triangular solid on the isometric grid paper below using your own measurement.



3. Study the following solid. Draw its top view, side view and front view on the squared paper using 5 units long and 4 units wide.



4. Which of the figures represents the front view of the solid?



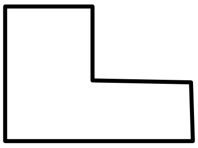


Figure 1

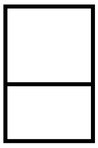


Figure 2

Figure 3

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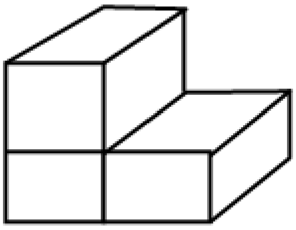
5. Which of the figures represents the side view of the solid?



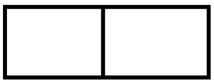


Figure 1

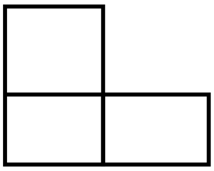


Figure 2

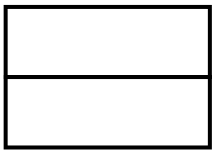


Figure 3

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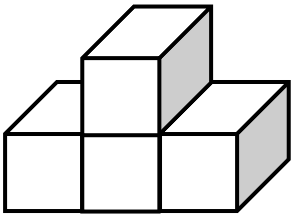
6. Which is the top view of the solid?



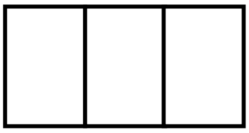


Figure 1

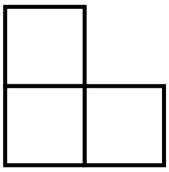


Figure 2

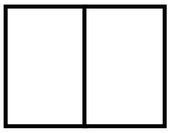


Figure 3

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 4

Lesson 15: Construction of Triangles

You learnt to identify methods of drawing solid objects in the last lesson. You also learnt to construct a solid object using a grid paper.

In this lesson you will:

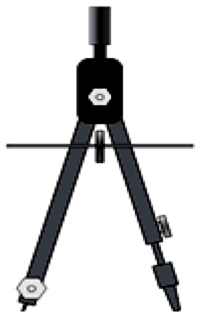
- Identify the steps in drawing an equilateral and an isosceles triangle using compass and a pencil.
- draw different types of triangles.

First let us define the term "**Construction**".

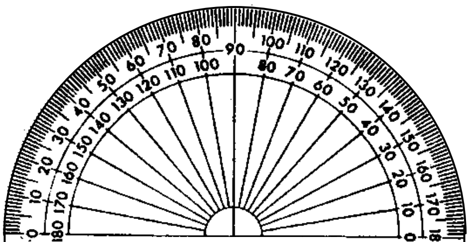
Geometrically and mathematically speaking

Construction means to draw shapes, angles or lines accurately.

These constructions use only compass, straightedge (i.e. ruler) and a pencil.

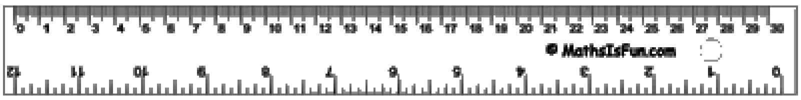


Compass



Protractor

Pencil



Straightedge or Ruler

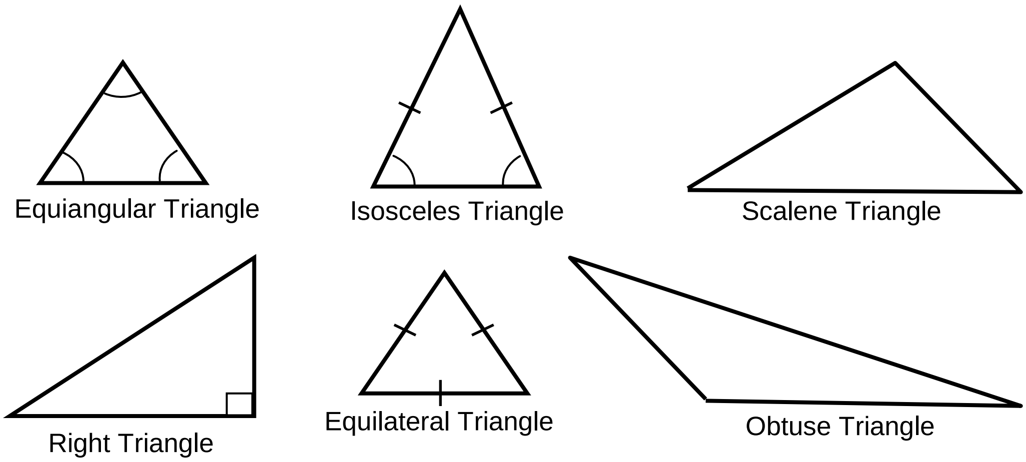
The basic constructions of the geometric shapes involve some steps that are to be followed. In this section, let us see how to construct a triangle.

A triangle is the most basic and simplest of all geometric figures

A triangle is a closed three sided figure. It is a polygon of three sides.

On the next page are pictures of the different types of triangle.

Here are the different types of triangle.



How to construct a Triangle

We can construct a triangle with the help of a ruler, a compass and a protractor. Since a triangle has three sides, it has three angles.

To draw a triangle, we must be given certain information about the lengths of its sides and the sizes of its angles.

If the measurements of three sides, the measurements of two sides and the angle between them, or the measurements of two angles and the side between them are given, then a triangle can be constructed by using a ruler, a protractor and a compass.

1. Constructing a Triangle given Three Sides

Example 1

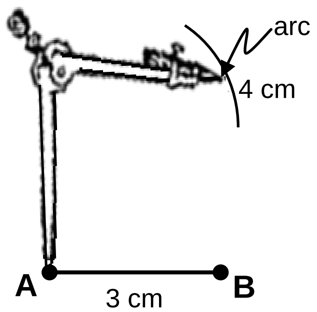
Using a ruler and <u>compass</u>, construct a triangle *ABC* with *AB* = 3 cm, *BC* = 4 cm and *AC* = 5 cm.

Solution:

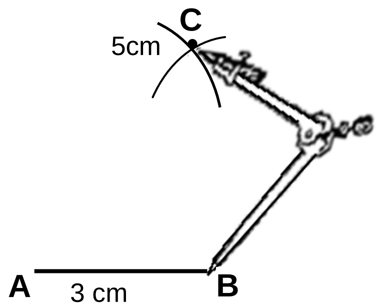
Step 1**:** Draw a line, *AB*, 3 cm long. Mark the endpoints.

Step 2: Draw an <u>arc</u> of <u>radius</u> 4 cm with **A** as the centre.

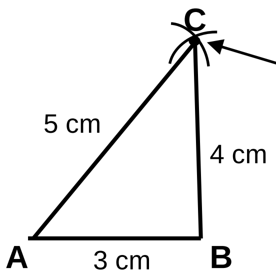
Note: Figures are not drawn to scale.



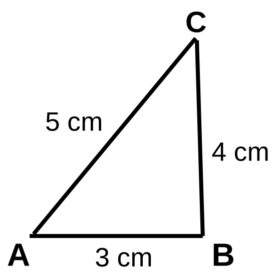
Step 3: Draw an arc of radius 5 cm with **B** as the centre to cut the arc drawn in Step 2 at **C**.



Step 4: Join, **C**, the point of intersection of the two arcs to the points **A** and **B.**



Step 5: Erase the arcs to obtain the required triangle **ABC.**



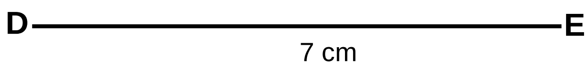
2. Constructing a Triangle given Two Angles and the Side between Them

Example 2

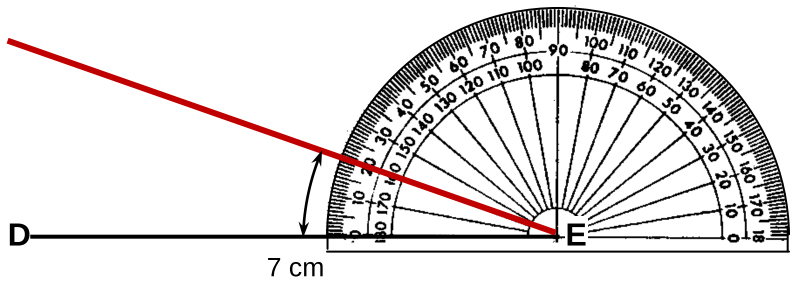
Using a ruler and a protractor, construct a triangle DEF with DE = 7 cm, $\angle$DEF = 25º and $\angle$EDF = 60º.

Solution:

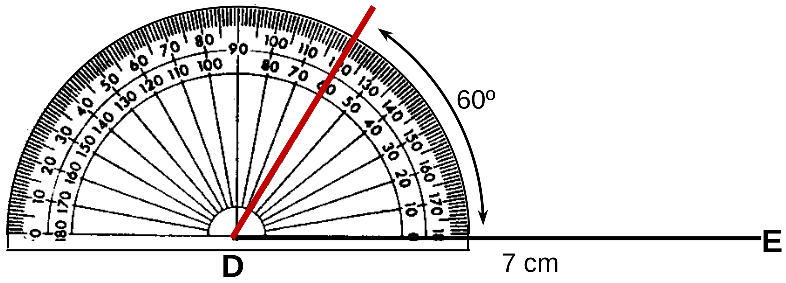
Step 1: Draw a line DE, 7 cm long.



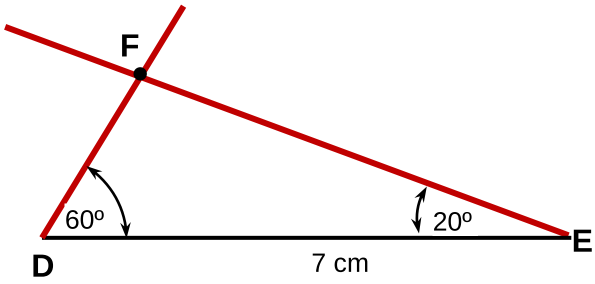
Step 2: Use a protractor to draw an angle, $\angle$DEF = 25º.



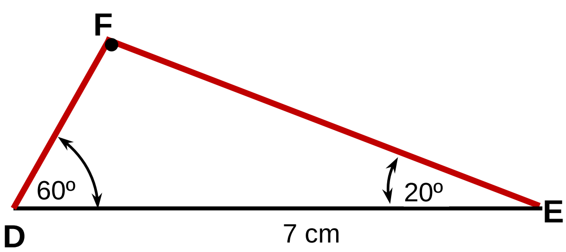
Step 3: Draw $\angle$EDF = 60º.



Step 4: Extend the line until they intersect each other at F. to form a triangle.



Step 5: Erase any extra lengths to obtain the required triangle.



3. Constructing a Triangle given Two Sides and an Angle

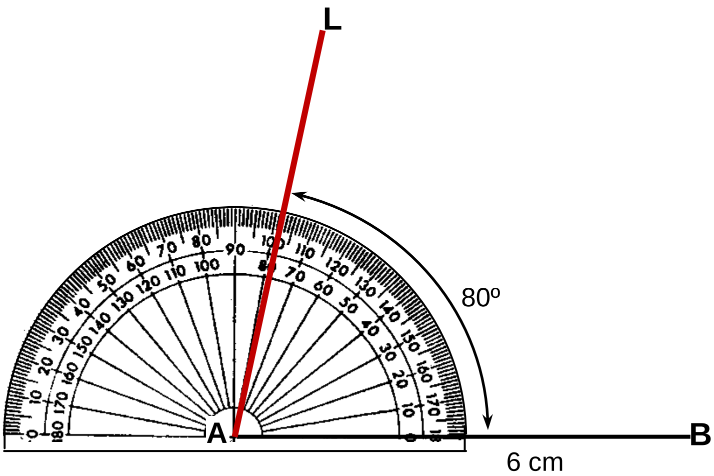
Example 3

Use a ruler, protractor and a compass to construct a triangle ABC with AB = 6 cm, BC = 7 cm and $\angle$BAC = 80º.

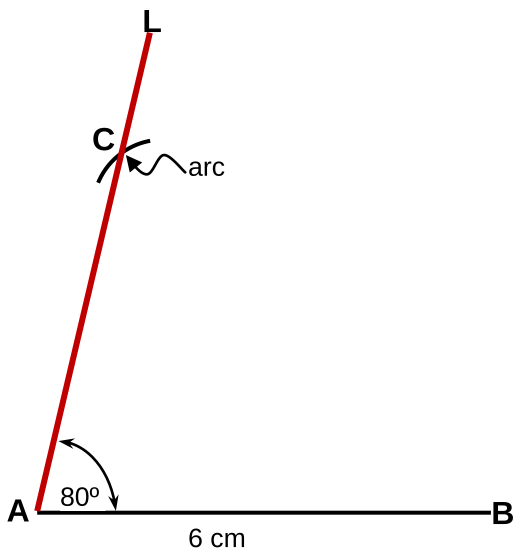
Solution:

Step 1: Draw a line AB, 6cm long.

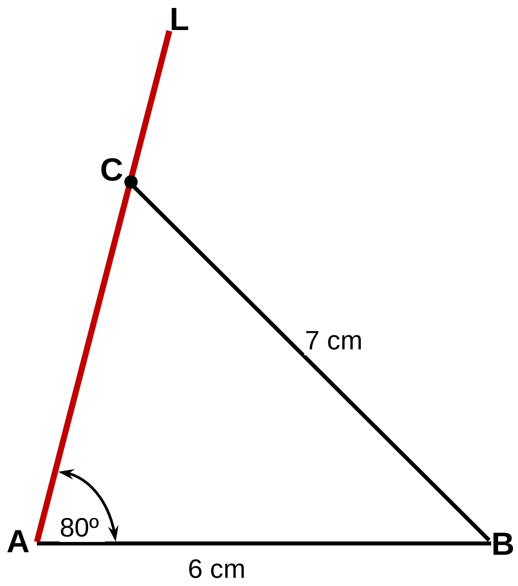
Step 2: Draw $\angle$BAL = 80º.



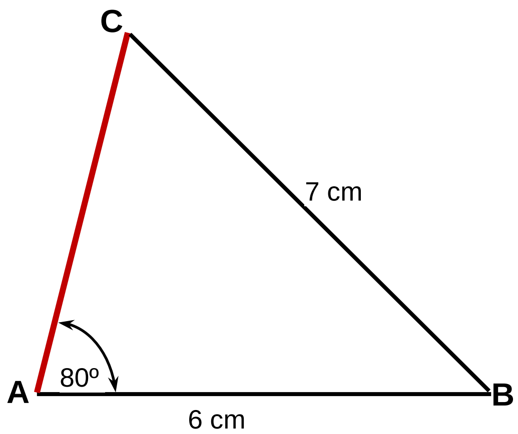
Step 3: Draw an arc of radius 7 cm with B as the centre to cut the arm AL of $\angle$BAL at C.



Step 4: Join B and C.



Step 5: Erase the arc and the extra length CL to obtain the required triangle ABC.



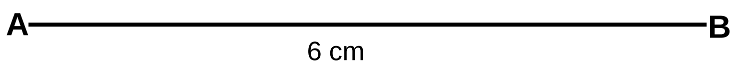
4. Constructing a Triangle given Two Sides and the Angle between Them

Example

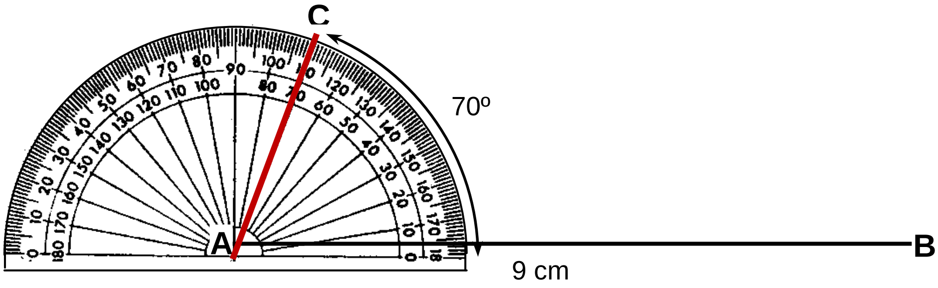
Using a ruler and a protractor, construct a triangle ABC with AB = 9 cm, AC = 7 cm and $\angle$BAC = 70º.

Solution:

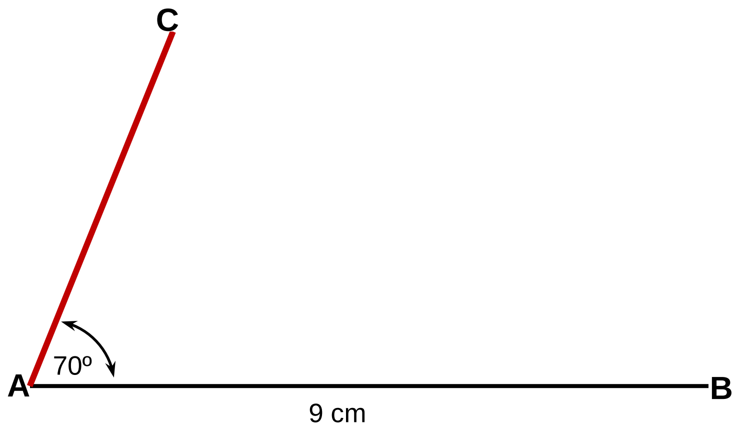
Step 1: Draw a line AB, 9 cm long.



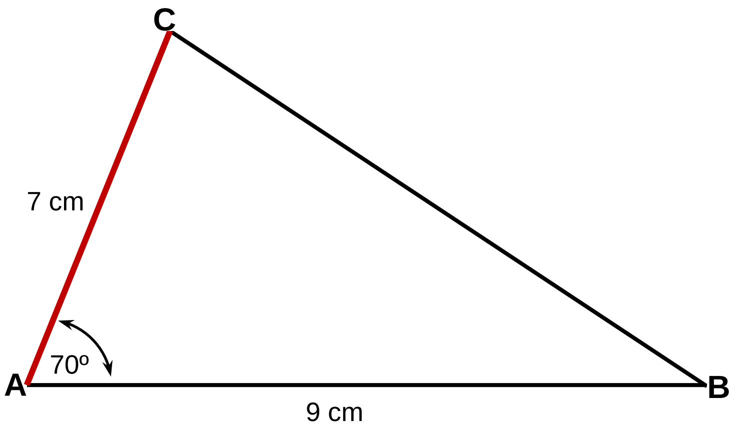
Step 2: Mark an angle of 70º by placing the centre of the protractor at the point A.



Step 3: Join 70º mark and the point A. Extend the arm AC until it is 7 cm long.



Step 4: Join the points B and C to obtain the required triangle ABC.



What we have drawn in the different examples are triangles of unequal sides and unequal angels.

On the next page, we will construct two special triangles, the equilateral triangle and the isosceles triangle.

- **An equilateral triangle is a triangle with all its three sides equal.**
- **An isosceles triangle is a triangle with two sides equal. The third side is the base. The two angles adjacent to the base are equal.**

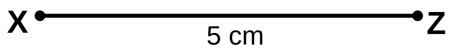
5. Constructing an Isosceles Triangle.

Example

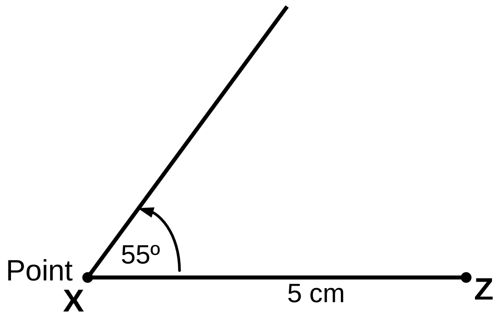
Using a ruler and a protractor, construct a triangle XYZ with XZ = 5 cm, ∠DEF = 55º and ∠EDF = 55º.

This can be done in several ways. Here is one way.

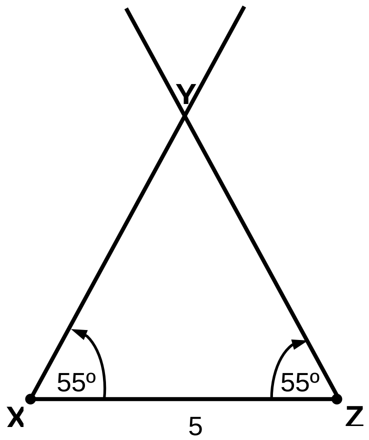
Step 1: Draw the side XZ 5 cm in length.



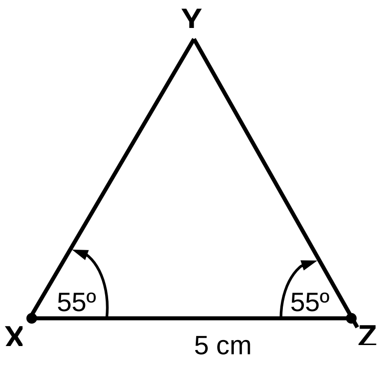
Step 2: Using one end of the line as the point for your first angle, draw an angle of 55º at X with your protractor.



Step 3: Using the other end of the 5 cm line, draw the second angle of 55º at Z with your protractor. Extend the arms to meet at Y.



Step 4: Erase any extra lengths to obtain the required triangle.



6. Constructing Equilateral Triangle

Example

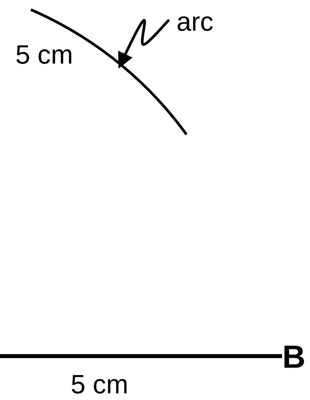
Using a ruler and a compass, construct an equilateral triangle ABC whose side length is 5 cm.

Solution:

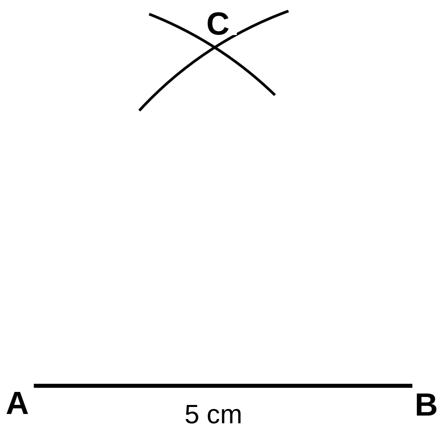
Equilateral triangles have all three sides the same length. This means that all three angles are also identical.

Step 1: Draw a line AB = 5 cm that is to be the first side of the triangle.

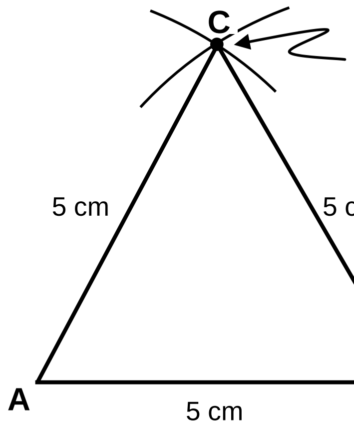
Step 2: Set your compasses to the length of the first side, position them at one end of the line AB, and draw an arc.



Step 3: Move your compass to the other end of the line, and draw an arc that crosses the first.

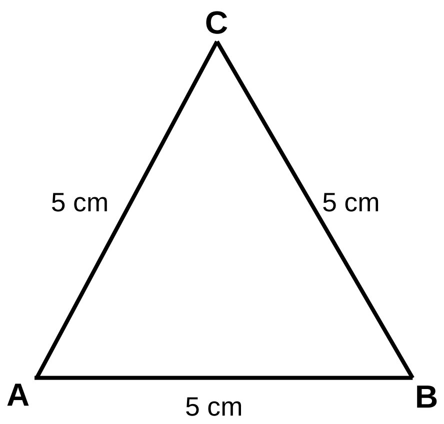


Step 4: The cross arcs mark the final point of the triangle. Join this point to each end point of the first line.



This point is 5 cm from one endpoint and 5 cm from the other endpoint of the line below.

Step 5: Erase the arc to obtain the required equilateral triangle.



NOW DO PRACTICE EXERCISE 15

Practice Exercise 15

1. Use a protractor, compass and ruler to construct each of the following triangles.

   (a) Triangle ABC with AB = 8 cm, $\angle$ABC = 40º and $\angle$BAC = 54º

   (b) Triangle ABC with AB = 10 cm, BC = 8 cm and AC = 6 cm

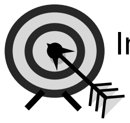
(c) Triangle ABC with AB = 6 cm, $\angle$ABC = 85º and BC = 7 cm.

(d) Triangle ABC with AB = 7 cm, AC = 5 cm and $\angle$BAC = 54º

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 4.

Lesson 16: Line Bisector

You learnt how to construct the different types of triangles in Lesson 15 using compass, protractor, ruler and pencil.



In this lesson, you will:

- define a line bisector
- identify the steps in constructing a line bisector a pair of compass, a straightedge or ruler and a pencil
- apply the steps in constructing a line bisector using a pair of compass, a straightedge or ruler and a pencil.

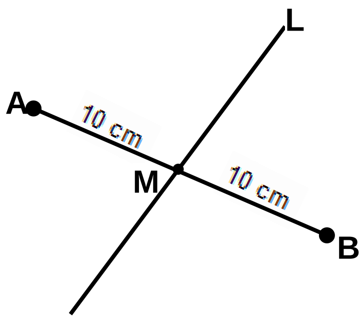
First let us define what a line bisector means.

A line bisector is a line, a ray or a segment which cuts another line segment into two equal parts.

To bisect something means to cut into two equal parts. The "bisector" is the one doing the cutting.

With a line bisector, we are cutting a line segment into two equal lengths with another line.

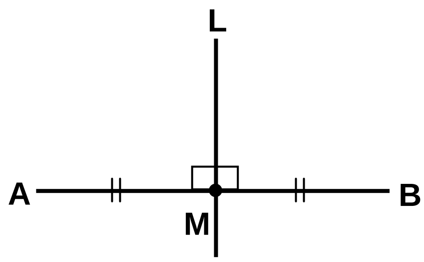
For example, look at the figure.



In the figure above, line **L** is the bisector of segment **AB** at point **M.** The line **AB** being cut into two equal lengths **AM** and **MB** by the bisector line **L.**

Point **M** is called the midpoint of the line **AB**.

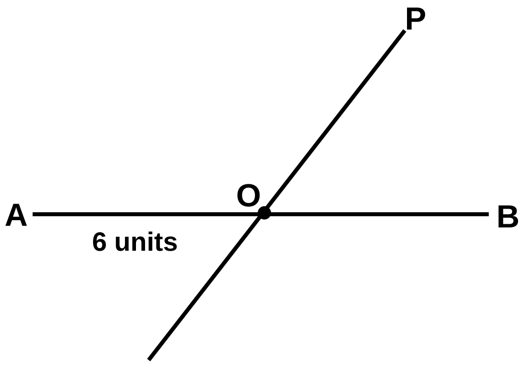
If Line **L** crosses at a right angle (90º), it is called the **perpendicular bisector**, of line **AB.** If it crosses at any other angle, it is simply called a **bisector.**



Let us solve an example of problem on line bisector.

Example 1

What is the length of AB, if the line P is the segment bisector and AO = 6 units.



Solution:

Step 1: Since Line P is the segment bisector of line AB, it divides AB into two equal parts. **O** is the midpoint of **AB**, since a segment bisector is a line passing through the midpoint of the segment.

Step 2: **AB** = 2(AO)

= 2(6 units) Substitute **AO** = 6 units

= 12 units

Step 3: The length of **AB** is 12 units.

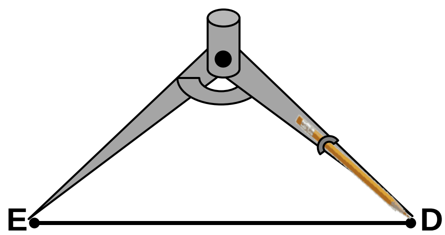
One way to bisect a line segment is to measure its length, divide by two and mark the midpoint.

But we can also do it without measurement at all using just a compass and a straightedge or ruler.

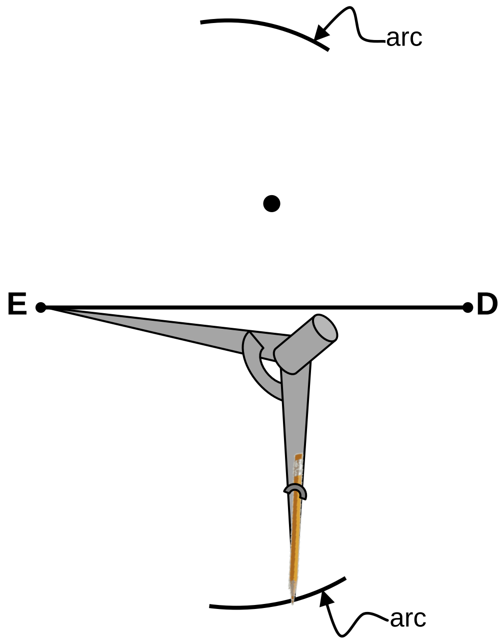
Here is how we bisect a line.

Step 1: Start with line **ED**.

Step 2: Set the compass to a distance about the length of ED.

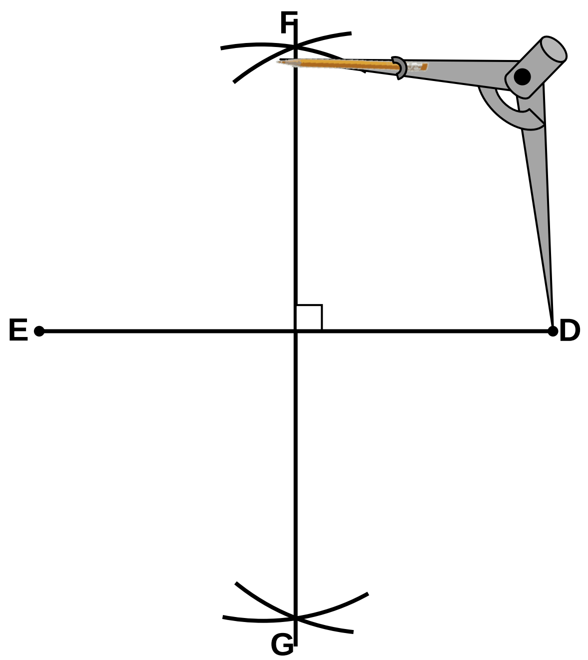


3. Place the compass point at **E** and draw an arc above **ED** and another arc below **ED**.



4. Using the same compass setting, place the compass at **D**. Draw arcs to intersect the first arcs. Label the intersections **F** and **G**.

   Join **FG.** The line **FG** bisects **ED**.



FG is at right angle to **ED**.

You must show your construction lines clearly. Do not rub them out.

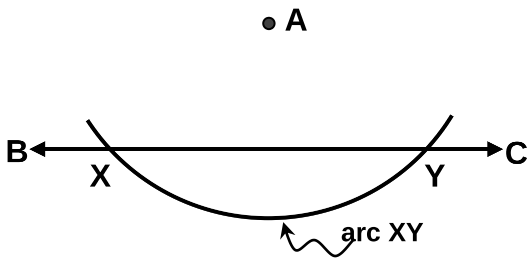
We can also construct a perpendicular bisector from a point to a line.

See next page.

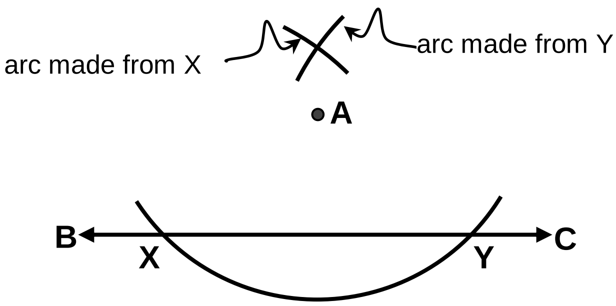
To construct a perpendicular bisector from point **A** to a line **BC:**

Step 1: Start with line BC.

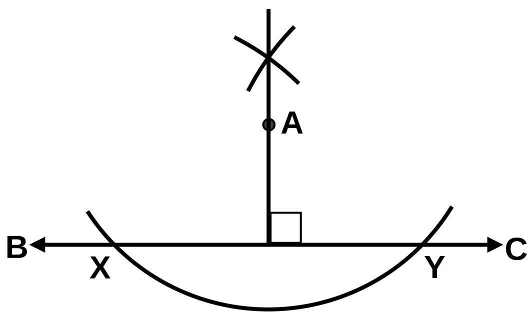
Step 2: With centre A draw an arc that cuts the line **BC** twice at **X** and **Y**.



Step 3: With centre **X** open the compass over half the length of the line segment **XY** and make an arc. With the same radius and centre **Y** make another arc crossing the arc made from **X.**



Step 4: Join the crossing point of the arcs and **A,** extending the line through **BC**.



Step 5: The line is perpendicular to **BC**.

Note:

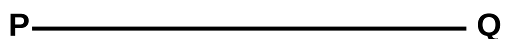
A line segment is a measureable portion of a line. It is finite in length. Straight lines can be infinite

NOW DO PRACTICE EXERCISE 16

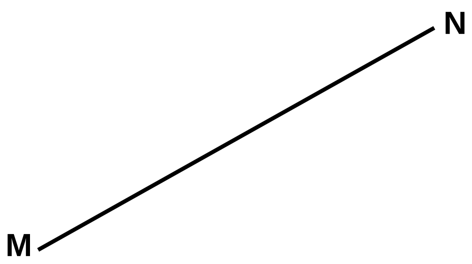
Practice Exercise 16

1. Bisect the lines below by following the instruction given on pages 155 and 156.

   a) Line **PQ.**

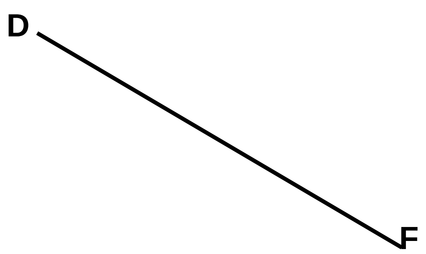


   b) Line **MN**.



   c) Line **VW.**

   d) Line **DF.**



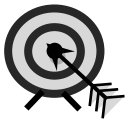
2. (a) Draw a line **AB**, 6 cm long. Construct the perpendicular bisector of **AB**.

(b) Mark any point on the bisector and measure its distance to **A** and **B**. What can you say about the distance of any point on the bisector from **A** and from **B**?

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 4.

Lesson 17: Angle Bisector

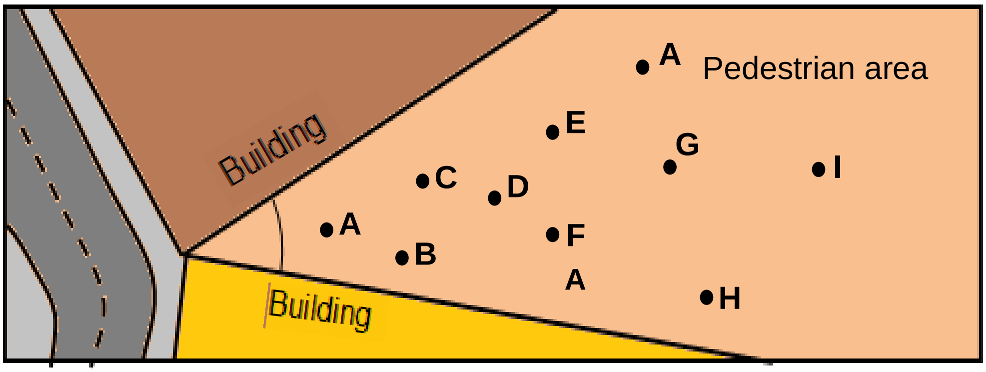
You defined a line bisector and learnt to construct a line bisector using a pair of compass in the last lesson.



In this lesson you will:

- define an angle bisector
- identify the steps in constructing an angle bisector using a pair of compass, a straightedge or ruler and a pencil
- apply the steps in constructing an angle bisector using a pair of compass, a straightedge or ruler and a pencil.

First, consider this picture and think ahead.



A pedestrian area is edged by two buildings which meet at an angle of 36º. The plan says trees must be planted at an equal distance from both buildings.

Which points on the diagram mark where trees could be planted?

This problem can be answered by connecting or joining all the point marks which are equidistant from both lines which meet at the angle of 36º. The term "**equidistant**" means "**the same distance from**". These points marked are the points **A**, **D** and **G**.

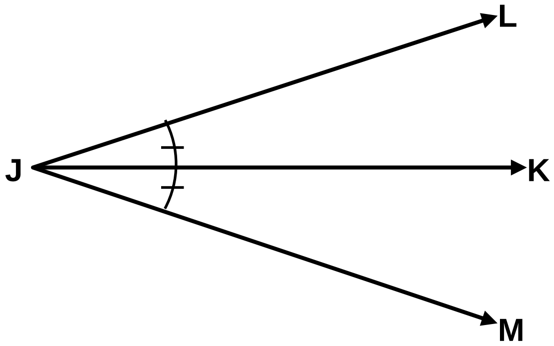
You will notice that the line drawn joining the Points **A, D** and **G** cuts the 36º angle in to two equal parts. We say, the line bisects the 36º angle.

As we learnt earlier, in general, to bisect something is to cut it into two equal parts. The bisector is the thing doing the cutting.

An angle bisector is a line which cuts an angle into two equal halves.

An angle bisector is a line passing through the vertex of the angle that cuts it into two equal smaller angles.

For example, the figure at the right, **JK** is the bisector. It divides the larger angle **LJM** into two smaller equal angles, $\angle$**LJK** and $\angle$**KJM.**



The two smaller angles are adjacent angles because they share the common vertex **J** and common side **JK.**

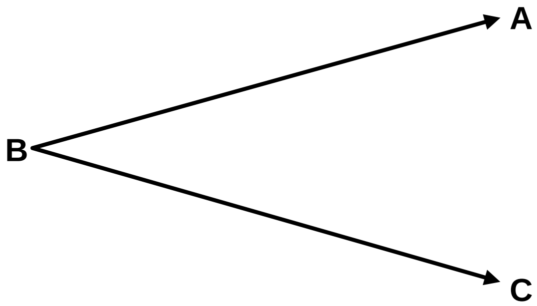
Bisecting an Angle

To bisect an angle means to draw a line which cuts it into two equal parts from the vertex.

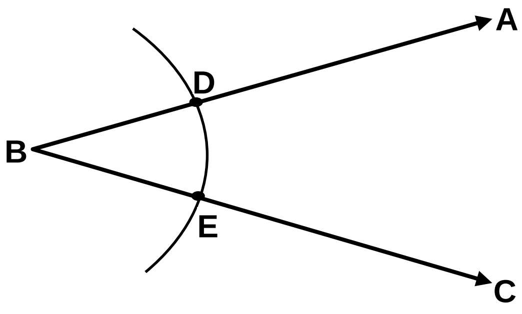
It would be useful to be able to bisect an angle without measuring it. You can do this by using a pair of compass, ruler and a pencil.

Here is how to bisect an angle..

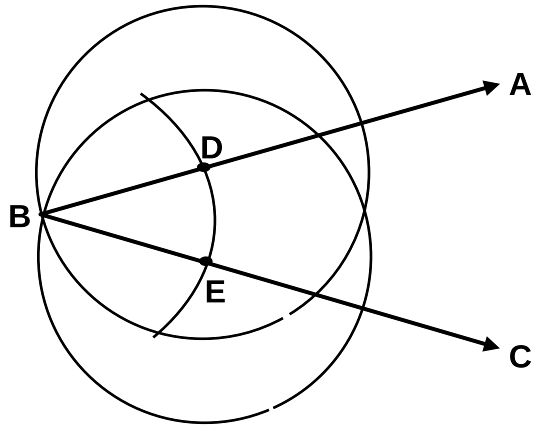
Step 1: Draw an angle **ABC**.



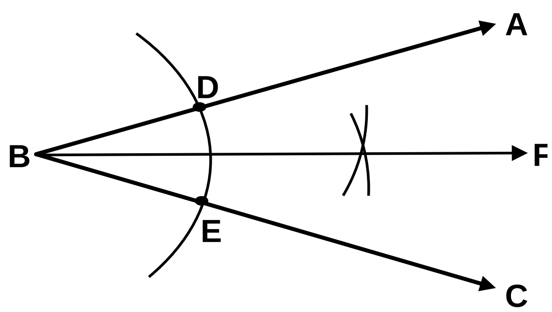
Step 2: With centre **B**, draw an arc so it cuts **AB** and **BC** at point **D** and **E**.



Step 3: With centre **D**, draw an arc.
With centre **E**, draw an arc.



Step 4: Draw the bisector **BF**. .



Use your protractor to check if $\angle$**ABF** is equal to $\angle$**CBF**.

NOW DO PRACTICE EXERCISE 17

Practice Exercise 17

1. Use a protractor to draw each angle below then bisect using compasses.

(a) 44°

(b) 100°

(c) 146°

(d) 90°

(e) 60°

(f) 120°

2. (a) Use compasses to draw an equilateral triangle with sides of 8 cm.

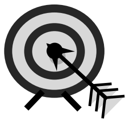
(b) What size is each angle of the triangle?

(c) Bisect one of the angles. What angle have you made?

CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 4.

Lesson 18: Constructing Regular Shapes

You defined the angle bisector and learnt to construct an angle bisector using a pair of compass in the last lesson.



In this lesson you will:

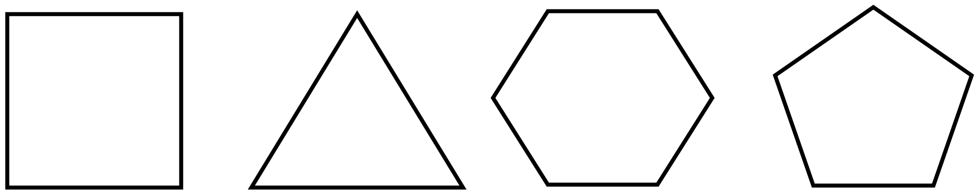
- define regular shapes
- draw regular shapes like squares, pentagon, hexagons and other regular shapes.

Many lessons in mathematics require the use of regular shapes such as equilateral triangles, squares and regular pentagons, hexagons and octagons.

While some of these shapes can be created with a protractor, it is often faster to create them with a compass and ruler.

Regular shapes are shapes that have all sides equal in length and all angles equal in size.

Below are examples of regular shapes.



We will use the properties of these shapes to help us draw each one of them.

First we have the square.

A square has the following properties:

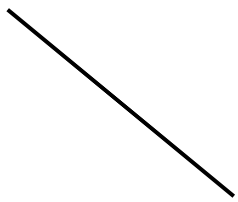
(i) 4 sides equal

(ii) 4 angles are equal (90º)

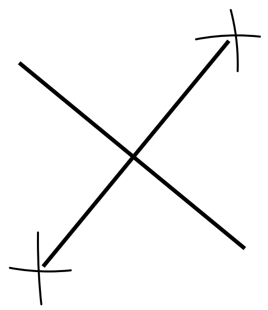
(iii) Diagonals bisect each other at 90º.

To draw a square, we use these properties using the following steps:

Step 1: Draw a line.

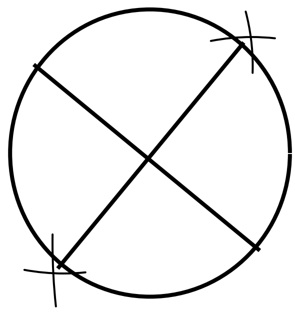


Step 2: Bisect the line.

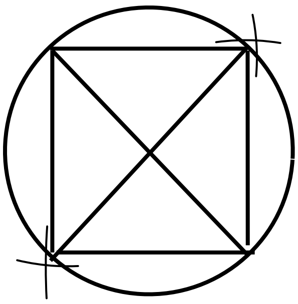


(You have now drawn the diagonals.)

Step 3: Set your compass to any distance. Place the compass point at the intersection of two lines and draw a circle.



Step 4: Use your ruler to join the sides of the square.



Next, you will learn to construct a regular octagon.

An octagon is a polygon of eight sides.

A regular octagon has the following properties:

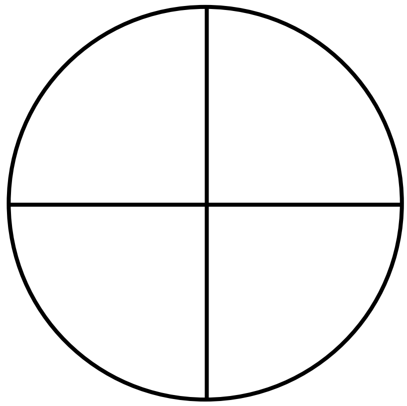
(i) Eight (8) equal sides

(ii) Eight (8) equal angles.

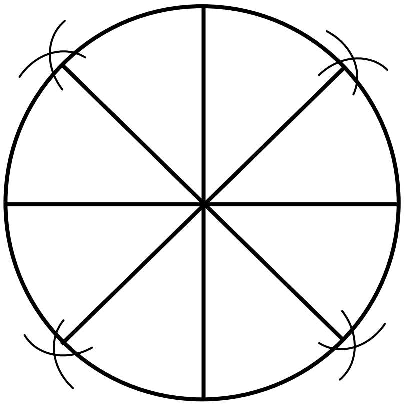
To draw a regular octagon, follow the steps 1 to 3 of drawing a square and then bisect the 90º angles formed at the intersection of the two lines.

See steps on the next page.

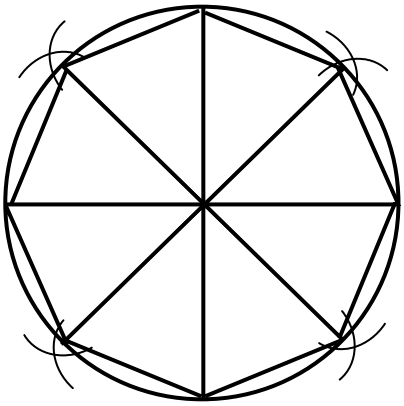
Step 1: Follow the steps 1 to 3 to draw a square.



Step 4: Bisect the angles. Draw the angle bisectors across the circle.



Step 5: Use your ruler to join the points on the circle.



Look at the other examples on the next page.

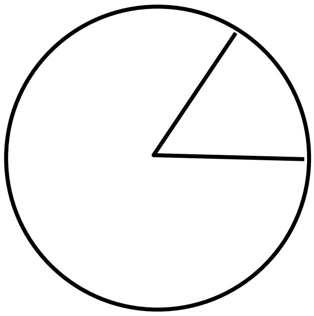
Example 1

Construct a regular hexagon.

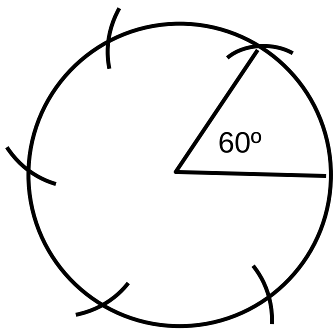
A regular hexagon has 6 equal sides and 6 equal angles.

To construct a regular polygon do the following steps:

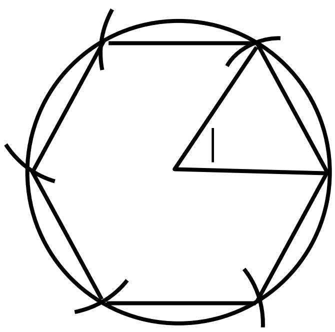
Step 1: Draw a circle and construct a 60º angle.



Step 2: Keeping the same radius of compass draw arcs on the circle placing the compass point on the previous arc.



Step 3: Use your ruler to join the intersections of arcs at the circle.



Example 2

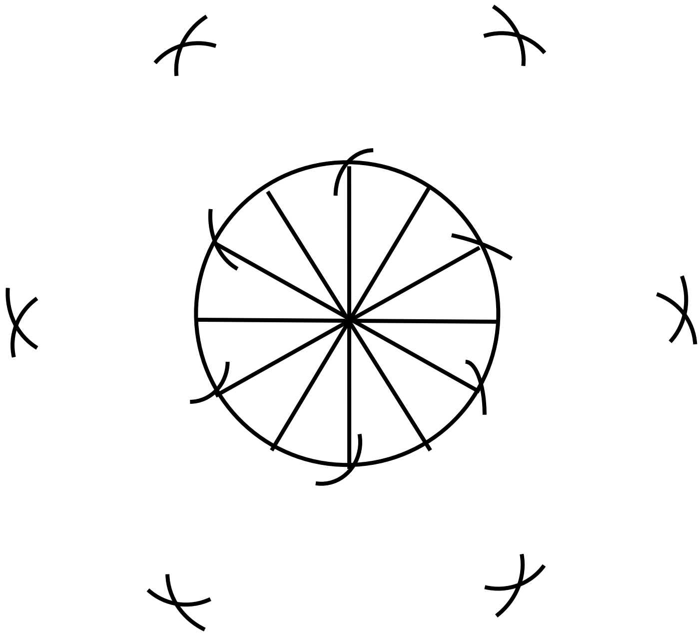
Construct a regular polygon with 12 sides.

A regular polygon of 12 sides is called a regular dodecagon.

To construct a regular dodecagon, follow steps 1 and 2 for drawing regular hexagons and then:

Step 3: Bisect the 60º angle.

Step 4: Use your ruler to join the points where the 60º arcs and the bisectors meet the circle.



NOW DO PRACTICE EXERCISE 18

Practice Exercise 18

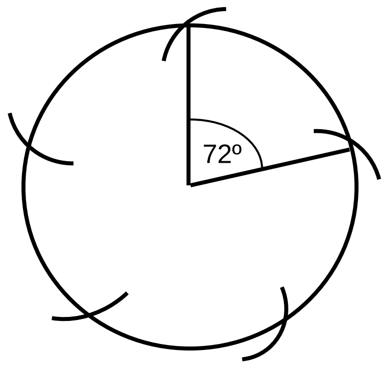
1. Using the circle drawn below and instructions that follow, construct a regular pentagon.

Instructions:

(a) Construct a 72º angle around the circle by marking 5 arcs around the circumference (part has been done for you).

(b) With a ruler join the:

First arc to the second arc, the second arc and the third arc, the third arc to the forth arc, and the forth arc to the fifth arc.



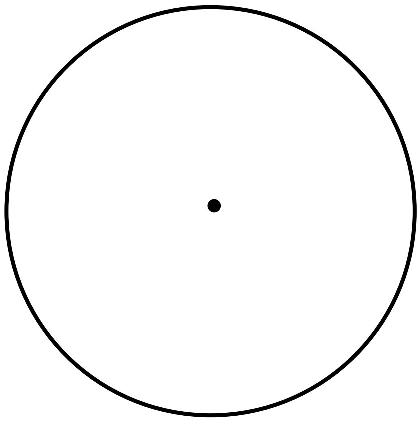
You have just constructed a regular pentagon.

2. Complete the statements below about a regular polygon with 12 sides. Then use these statements to help you construct the polygon.

   (a) If a twelve sided polygon is inscribed in a circle then the angle between the radii to two adjacent vertices of the polygon and the centre of the circle is found by dividing 360º by the number of sides, 12.

   That is: $\frac{360}{12}$ = ___________

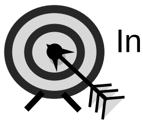
   (b) $\frac{60}{2}$ = ___________ so bisect 60º angles around the circle.



CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 4

Lesson 19: Nets

You defined regular shapes and learnt to construct different regular shapes using a pair of compass in the last lesson.



In this lesson you will:

- define nets
- identify nets of different figures
- draw nets of different solids
- make solids using their nets

Earlier in Grade 7 and 8 Mathematics, you learnt the different types of solid shapes.

When we are talking about solid figures, it is very helpful to have models to look at.

Solids can be made from plane shapes. This is done by drawing the net of the solid on a piece of paper. The net shows how the faces are joined to each other. When faces are folded along their edges, the solid is formed.

We make models of solid shapes to help us understand their properties using net.

The net of a solid shape is a two-dimensional representation that can be folded to produce a three-dimensional solid. It is the pattern it makes in two dimensions if we cut it out along edges and laid flat.

Let us begin this lesson, by looking at the examples of the nets of some solid figures of which you should make models. This includes prisms and pyramids such as a triangular prism, cuboids, square pyramids, cylinders and cones which you may already have made when you studied Upper Primary Mathematics.

Let us look again at the models of these solids and their nets.

Example 1 Triangular prism

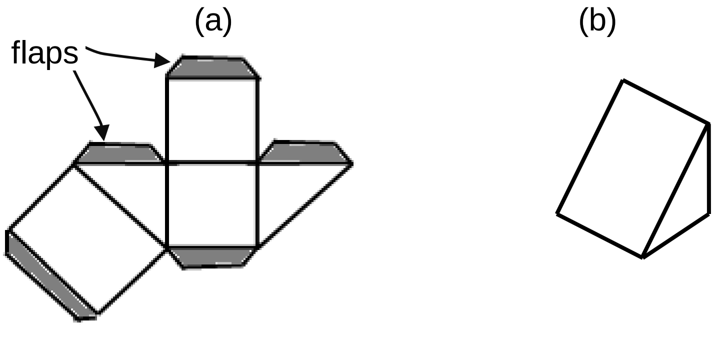
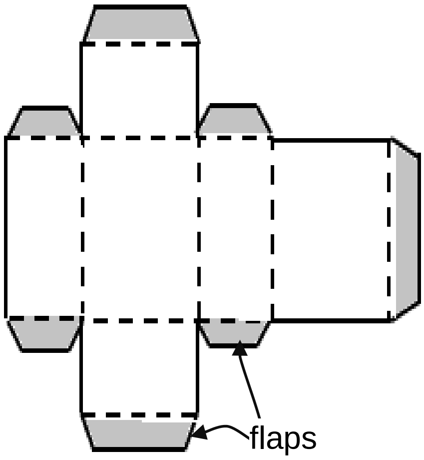


Figure 1

Figure 1(a) shows the net of a **triangular prism** and Figure 1(b) the completed model.

Example 2 Cuboid

(a)



(b)

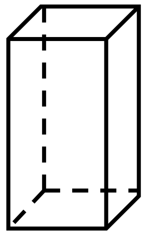
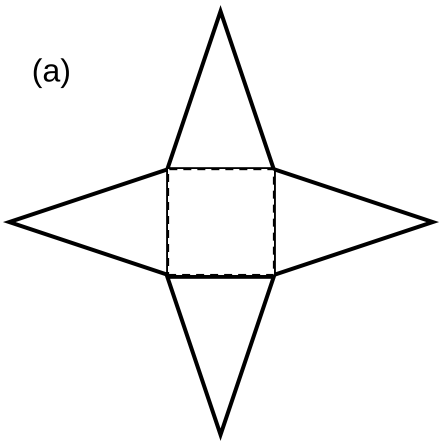


Figure 2

Figure 2(a) shows the net of a **cuboid** and Figure 2(b) the completed model.

Example 3 Square Pyramid

(a)



(b)

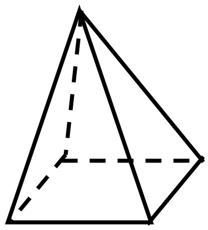
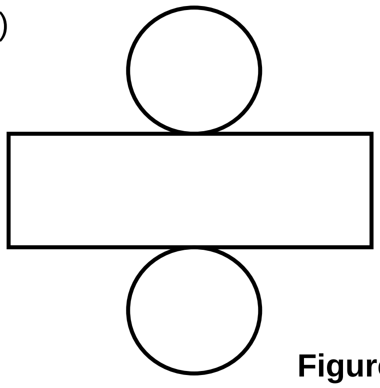


Figure 3

Figure 3(a) shows the net of a **square pyramid** and Figure 3(b) the completed model.

Example 4 Cylinder

(a)



(b)

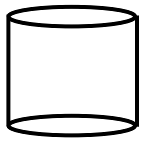
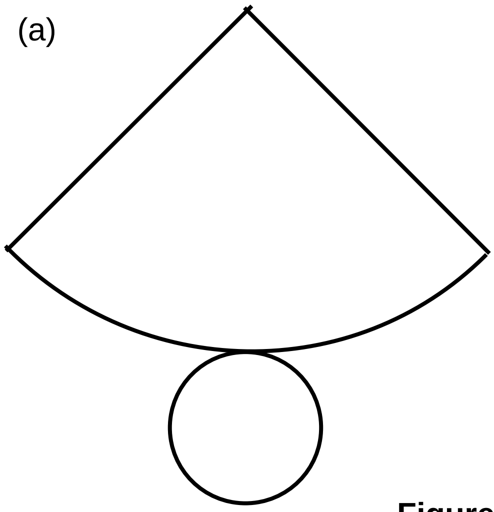


Figure 4

Figure 4(a) shows the net of a **cylinder** and Figure 4(b) the completed model.

Example 4 Cone

(a) (b)



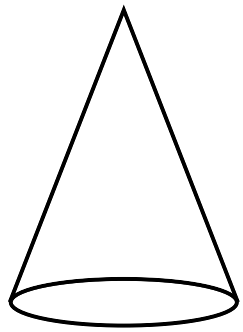


Figure 5

Figure 5(a) shows the net of a cone and Figure 5(b) the completed model.

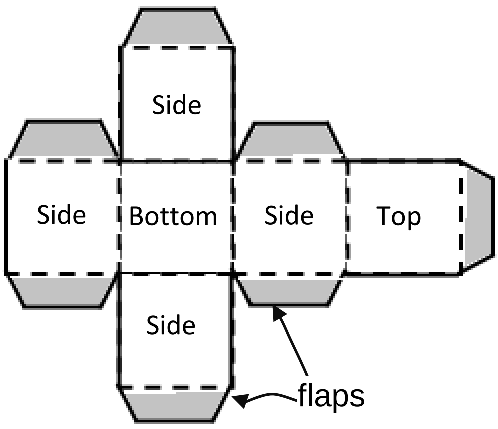
We make models of solid figures to help us understand their properties. We fold the nets to make models.

Some nets are quite easy to make while others are hard.

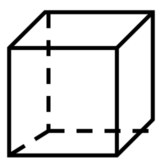
The cube is the easiest solid shape to think about. Its faces are all squares.

The easiest way to find a net is to think of a cube as having four sided sides, a top and a bottom.

Net of a Cube



Model of a Cube



We fold the nets to make models.

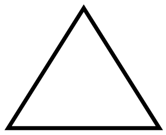
On the next page, are activities for you to do.

You will need the following:
1. Ruler
2. Protractor
3. Tape
4. Paper

Follow the directions to make each net. Then fold your nets into three-dimensional figures.

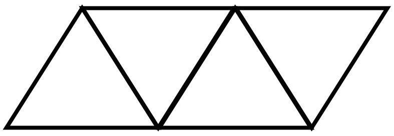
Activity # 1

Step 1: Draw a 2 cm equilateral triangle.
The measurement of each angle is 60º.



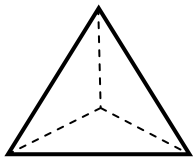
Step 2: Draw three more of them to look like Figure 1.
There will be four triangles.

Figure 1



Step 3: Join the common edges and tape them together.
This is the net of a **tetrahedron**.

Step 4: Fold your net into a three-dimensional figure.

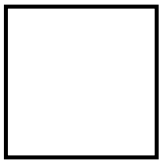


A tetrahedron is a regular polyhedron whose faces are four equilateral triangles.

Activity# 2

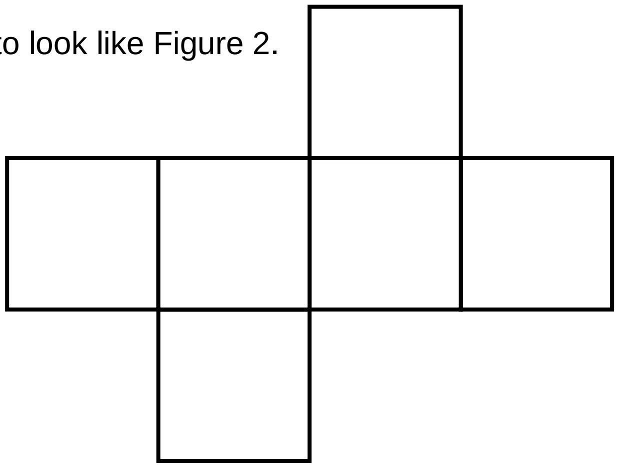
Follow the directions:

Step 1: Draw a 2 cm square.



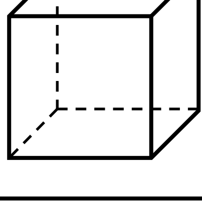
Step 2: Draw five (5) more of them to look like Figure 2.
There will be 6 squares.

Figure 2



Step 3: Join the common edges and tape them together.
This is the net of a **cube**.

Step 4: Fold your net into a three-dimensional figure.

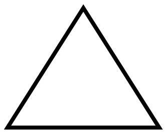


A cube is a regular polyhedron whose faces are all squares.

Activity # 3

Follow the directions:

Step 1: Draw a 2 cm equilateral triangle.



Step 2: Draw seven (7) more of them to look like Figure 3.

There will be 8 triangles.

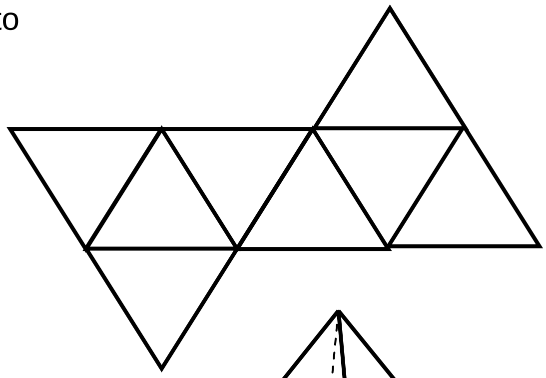
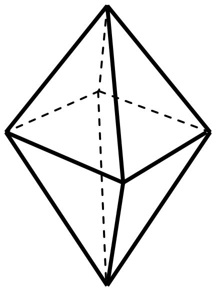


Figure 3



Step 3: Join the common edges and tape them together.
This is the net of an **octahedron**.

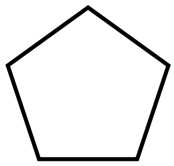
Step 4: Fold your net into a three-dimensional figure.

An octahedron is a regular polyhedron whose faces are eight equilateral triangles.

Activity # 4

Follow the directions.

Step 1: Draw a regular pentagon with side measuring 2 cm.
The measurement of each angle will be 108º.



Step 2: Draw eleven (11) more of them to look like Figure 4.
There will be 12 pentagons.

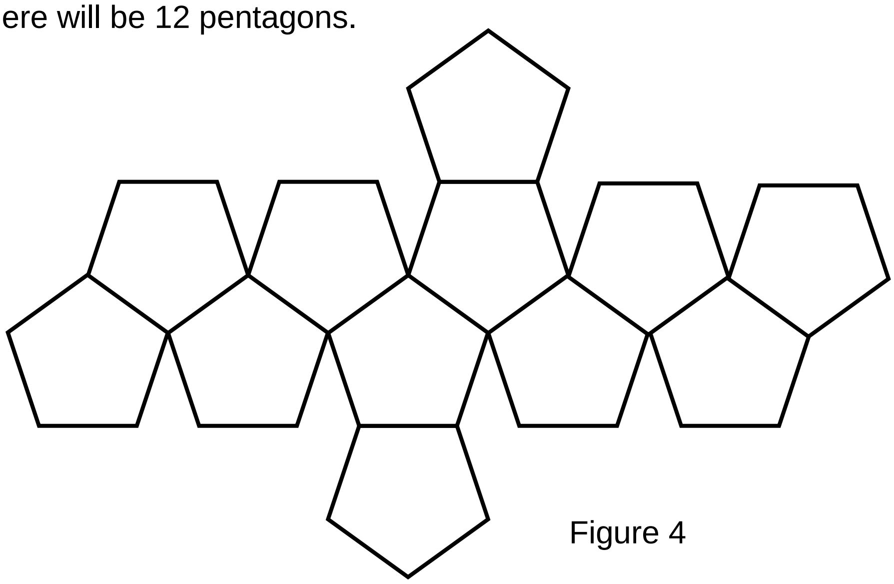
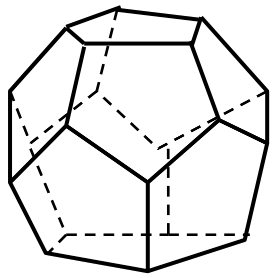


Figure 4

Step 3: Join the common edges and tape them together.
This is the net of a **dodecahedron**.

Step 4: Fold your net into a three-dimensional figure.



A dodecahedron is a regular polyhedron whose faces are twelve (12) regular pentagons..

What you have created in all these activities are models of regular solid figures in which every surface is a polygon.

As you know, a polygon is a closed plane figure formed by three or more line segments.

A solid figure whose faces are congruent polygons is called a polyhedron.

NOW DO PRACTICE EXERCISE 19

Practice Exercise 19

1. On the next pages you are given the nets to make models of solid.

   Follow the steps below.

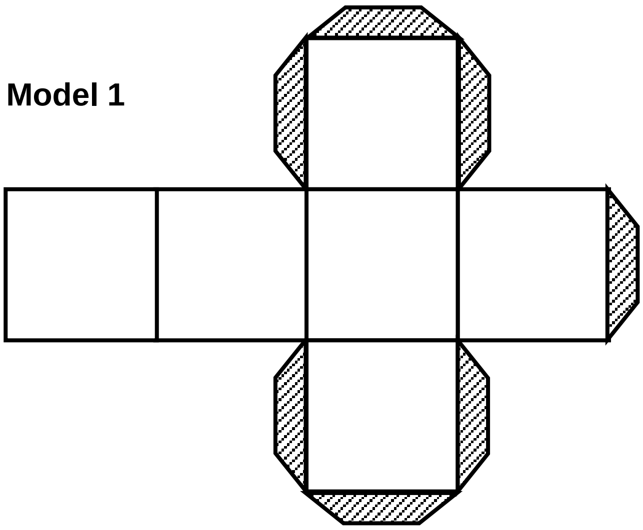
   Step 1: With your ruler and protractor, draw the pattern on a piece of heavy paper or card board.

   Step 2: Carefully cut out the pattern.

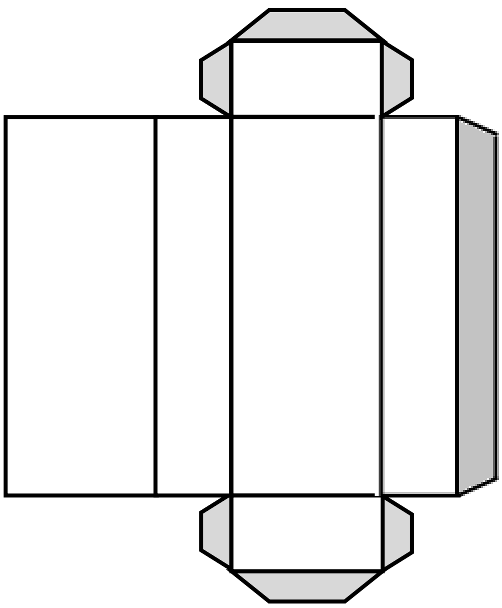
   Step 3: Fold where necessary and glue the flaps of the model together.

   Step 4: Write the name of the shape on the model.

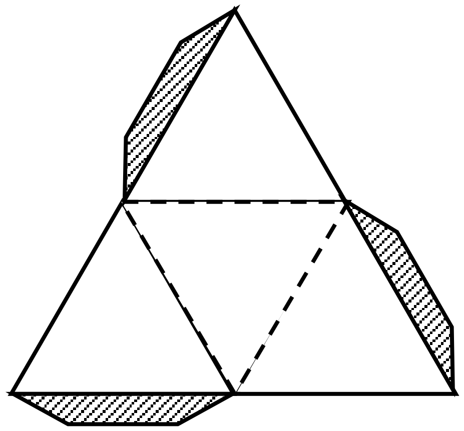
Model 1



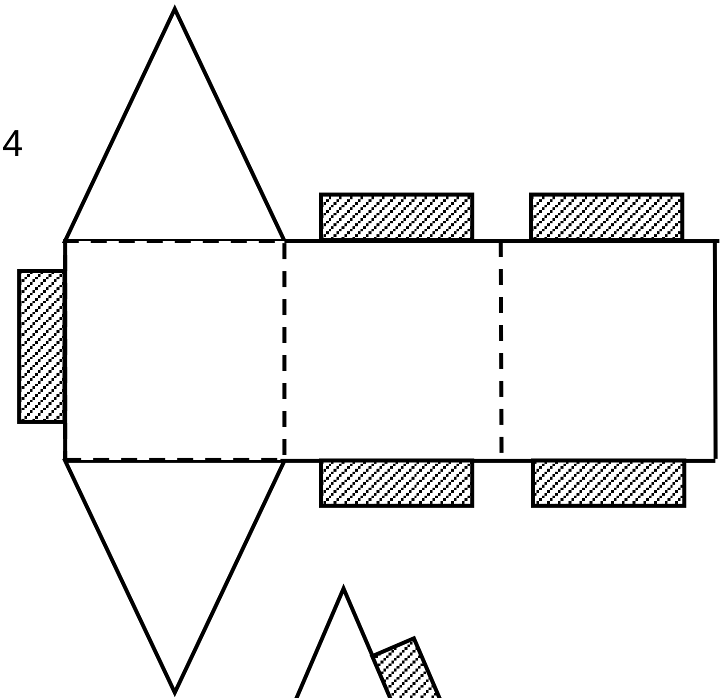
Model 2



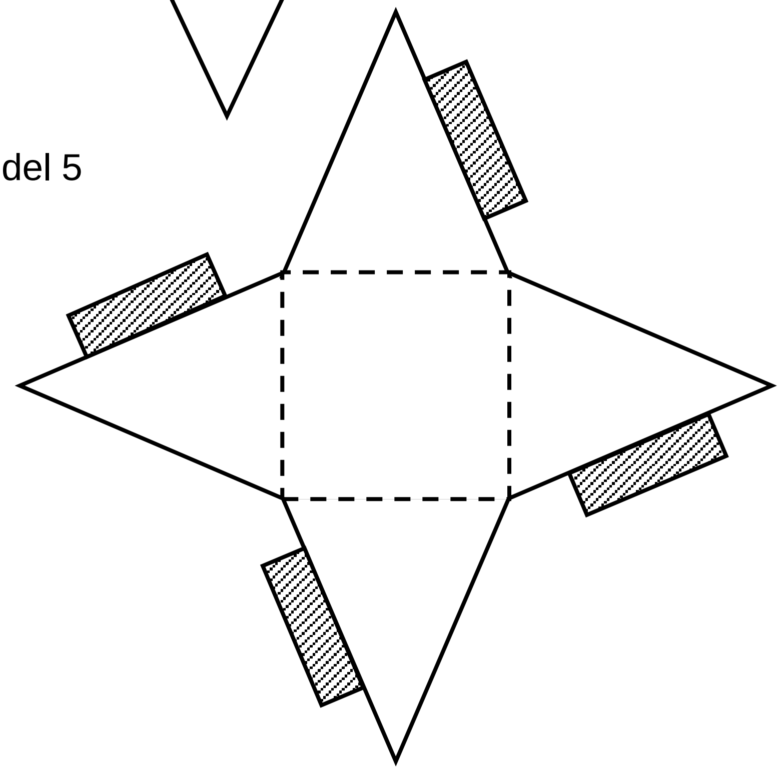
Model 3



Model 4



Model 5



2. (a) Study your models and then complete the table below.

Solid Shape	Number of Faces**(f)**	Number of Vertices(**v)**	Number of Edges(**e**)	f + v	e + 2
Cube	**6**	**8**	**12**	**14**	**14**
Cuboid					
Tetrahedron					
Triangular prism					
Square pyramid					

(b) Use the table to complete the following statements.

(i) For a Cube;

Number of faces + number of vertices = ___________

= ___________

Number of edges + 2 = __________

= __________

Number of faces + number of vertices = the number of __________+ __________

(ii) For a triangular Prism:

Faces + vertices = ___________

Edges + 2 = ___________

= ___________

Faces + vertices = ___________ + 2.

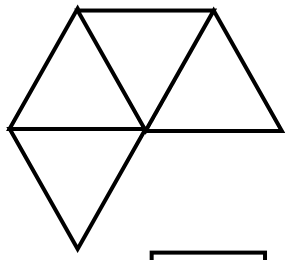
(c) Is the relationship f + v = e + 2 true for the following solids? Yes/No

(i) a Cuboid ________ (ii) a tetrahedron ________ (iii) a square pyramid _______

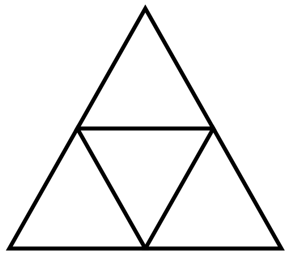
(d) The relationship between f, v, and e for these solids is _______________.

3. Copy and fold each net to determine whether it is the net of a polyhedron. If so, name the regular polyhedron that the net forms.

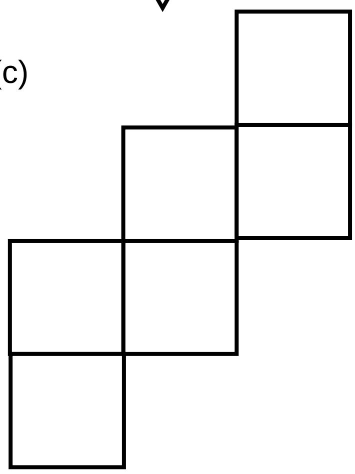
(a)



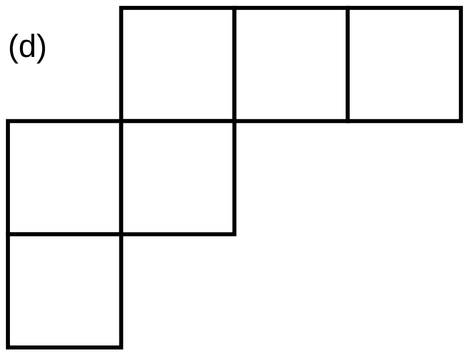
(b)



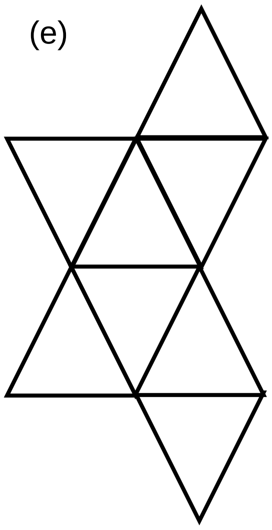
(c)



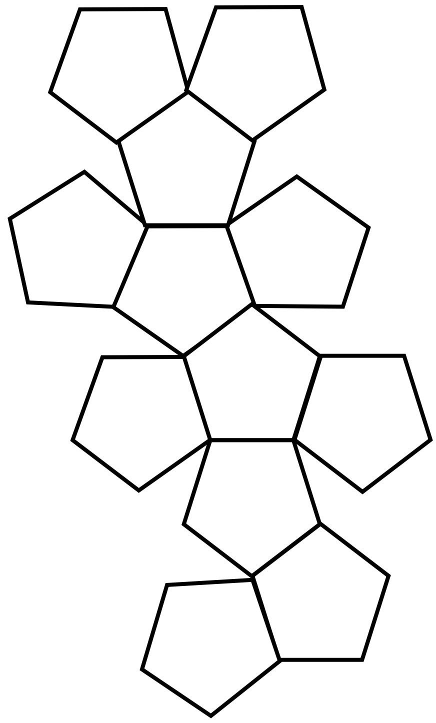
(d)



(e)

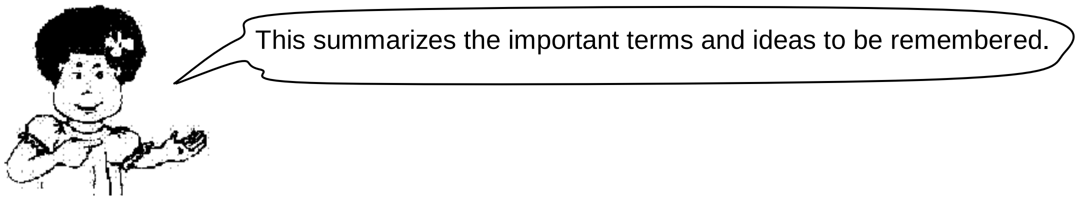


(f)



CORRECT YOUR WORK. ANSWERS ARE AT THE END OF TOPIC 4

TOPIC 4: SUMMARY



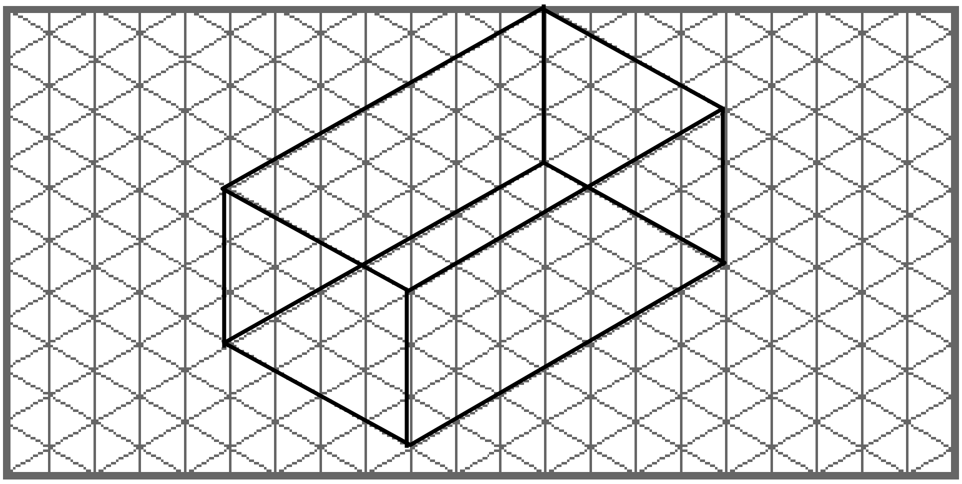
- **Construction** is drawing or creating various shapes using only compass and straightedge or ruler. No measurement of length or angles is required.
- When you **bisect** a figure, you **divide it into two congruent parts**.
- To **bisect a line**, we must **divide the line into two equal parts**. We can bisect without measuring the line.
- **A line bisector** is a line, ray or segment which cuts another line segment into two equal parts.
- To **bisect an angle**, we need to divide the angle into two equal smaller angles.
- An **angle bisector** is a line which cuts an angle into two equal halves.
- **Regular shapes** are shapes that have all sides equal in length and all angles equal in size.
- A **regular polygon** is a polygon that has equal sides and equal angles.
- A **polyhedron** is a solid figure whose faces are congruent polygons.
- The **net of a solid** shape is a two-dimensional representation that can be folded to produce a three-dimensional solid. It is the pattern formed in two dimensions if it is cut out along edges and laid flat.
- A **tetrahedron** is a regular polyhedron whose faces are four equilateral triangles.
- A **cube** is a regular polyhedron whose faces are all squares.
- An **octahedron** is a regular polyhedron whose faces are eight equilateral triangles.
- A **dodecahedron** is a polyhedron whose faces are twelve regular pentagons.

REVISE LESSONS 15 – 19 THEN DO TOPIC TEST 4 IN ASSIGNMENT 6

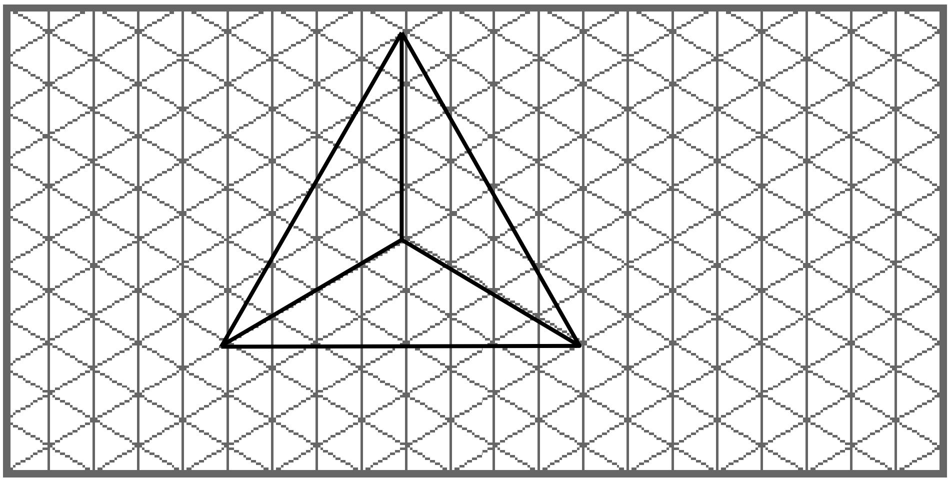
ANSWERS TO PRACTICE EXERCISES 14 -19

Practice Exercise 14

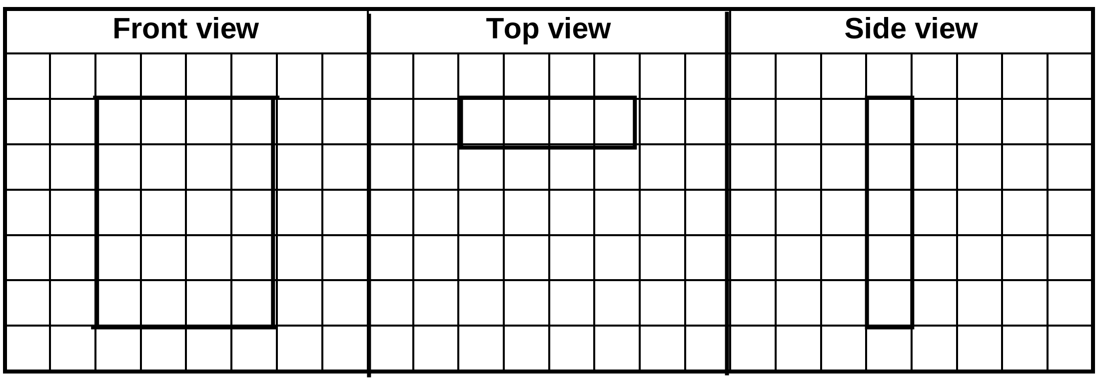
1.



2.



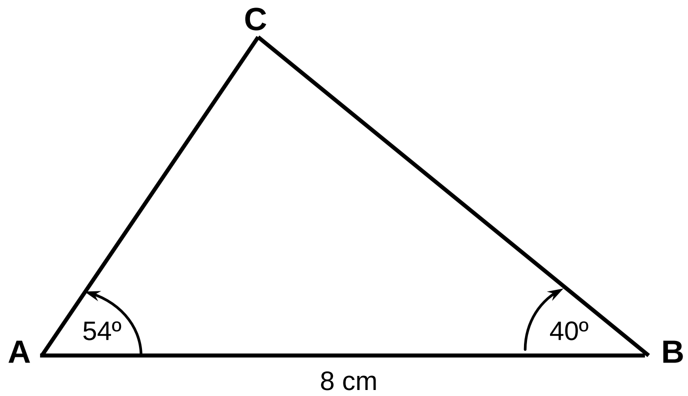
3.



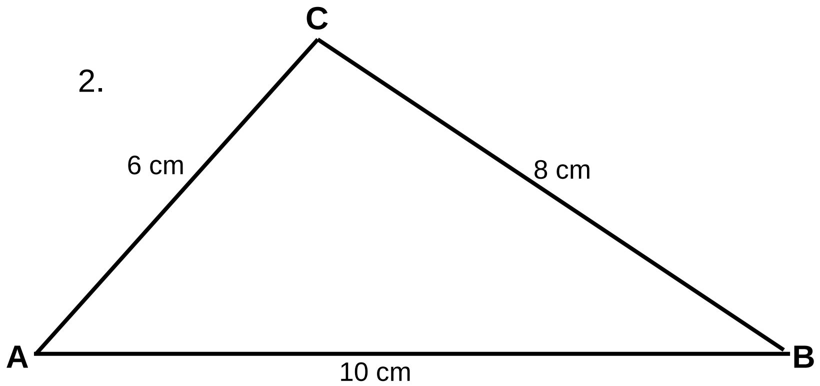
4. **Figure 1** 5. **Figure 3** 6. **Figure 1**

Practice Exercise 15

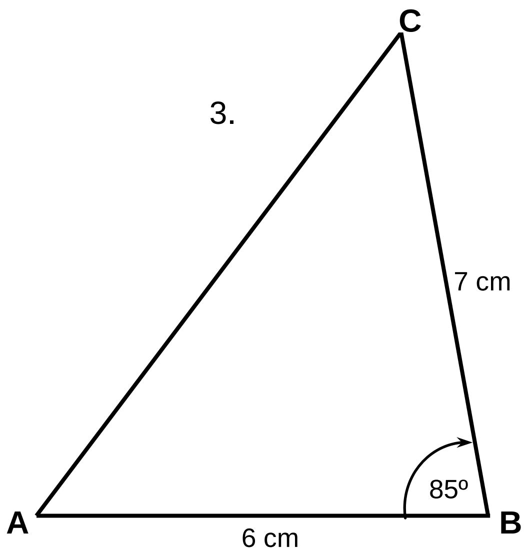
1.



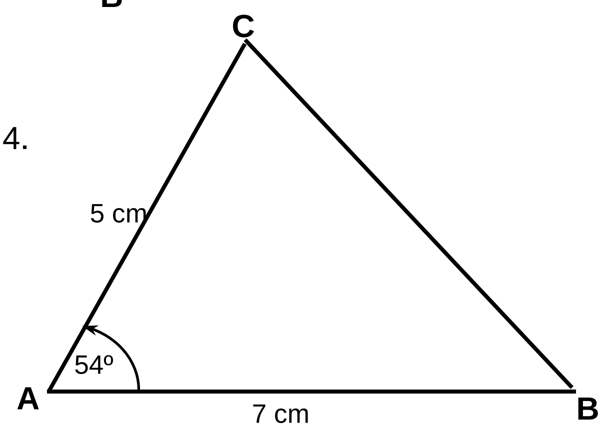
2.



3.

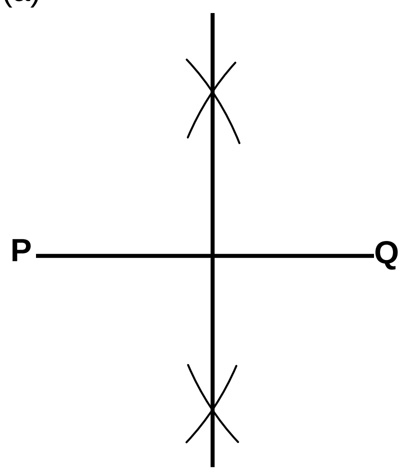


4.

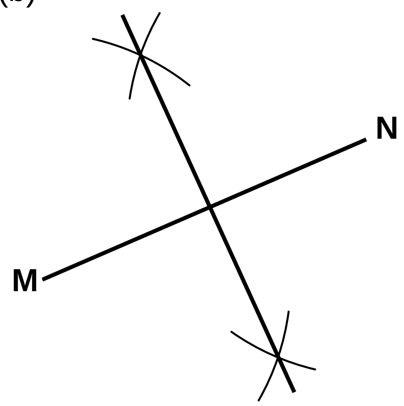


Practice Exercise 16

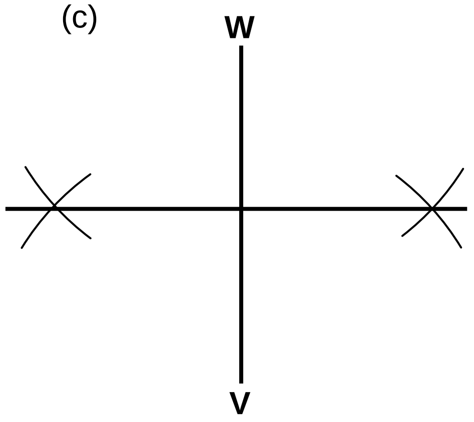
1. (a)



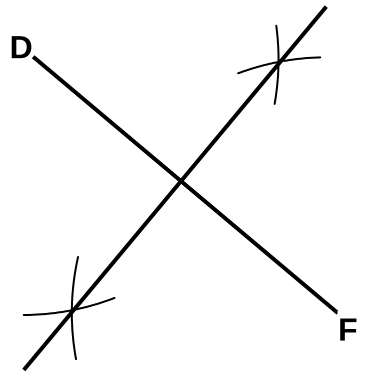
(b)



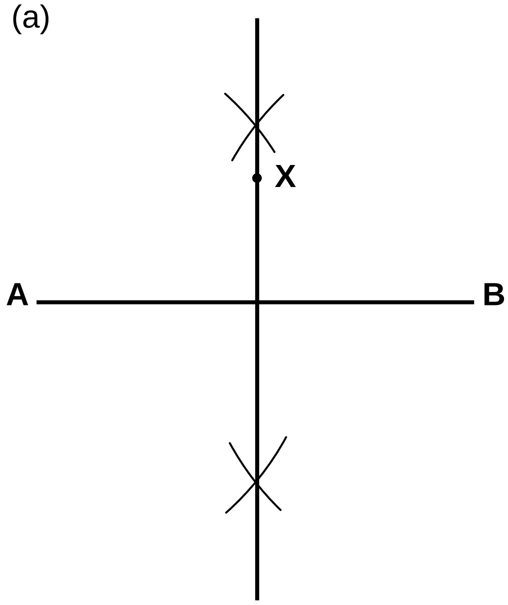
(c)



(d)



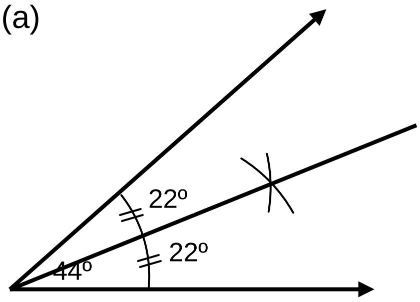
2. (a)



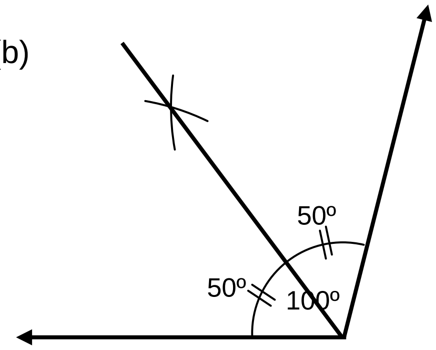
(b) The distance of any point (**X**) on the bisector from A and from B is the same.

Practice Exercise 17

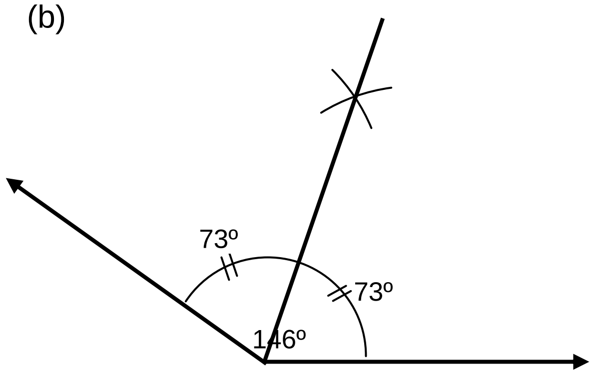
1. (a)



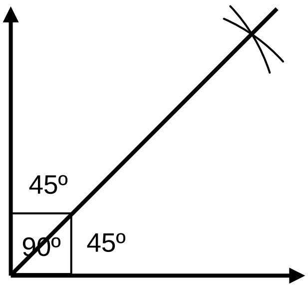
(b)



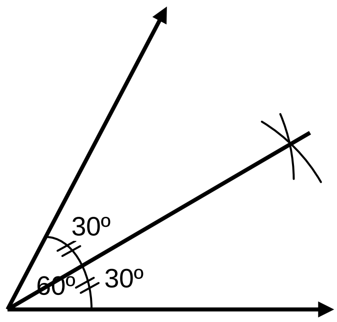
(b)



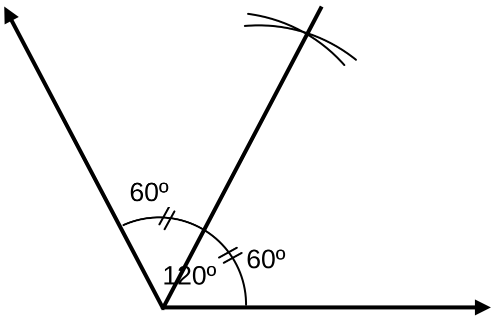
(d)



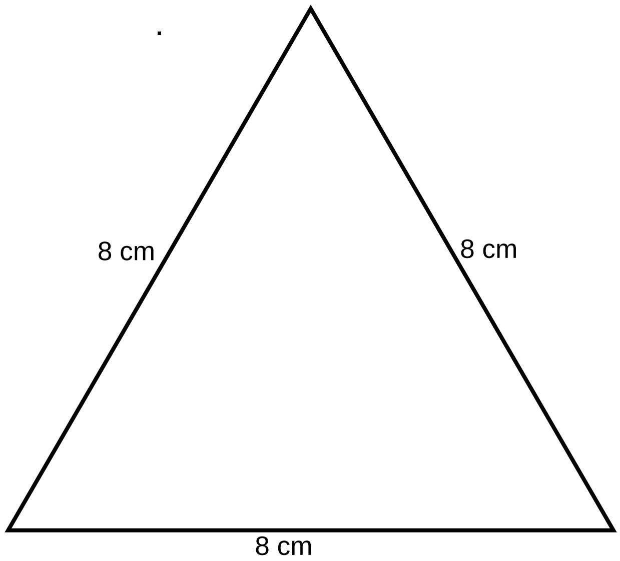
(e)



(f)



2. (a) .

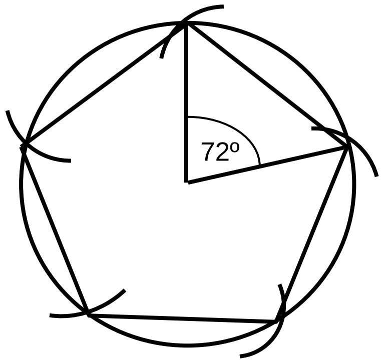


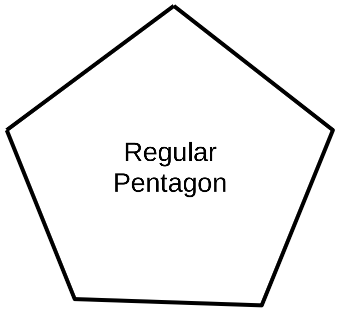
(b) Each angle is 60º

(c) The angle is a 30º angle.

Practice Exercise 18

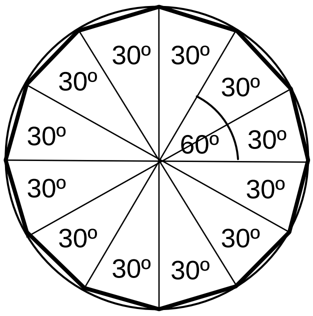
1.

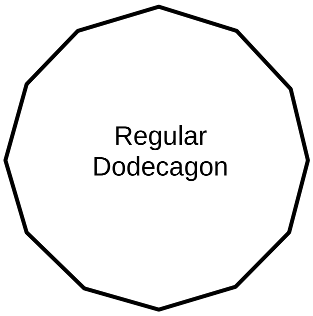




2. (a) 30º

(b) 30º





Practice Exercise 19

1. Model 1 = Cube
   Model 2 = Cuboid or Rectangular prism
   Model 3 = Tetrahedron
   Model 4 = Triangular prism
   Model 5 = Square Pyramid

2. (a)

Solid Shape	Number of Faces(f)	Number of vertices(v)	Number of edges(e)	f + v	e + 2
Cube	6	8	12	14	14
Cuboid	6	8	12	14	14
Tetrahedron	4	4	6	8	8
Triangular prism	5	6	9	11	11
Square pyramid	5	5	8	10	10

(b) i. For a cube

Number of faces + number of vertices = 6 + 8 = 14

Number of edges + 2 = 12 + 2 = 14

Number of faces + number of vertices = number of edges + 2

ii. For a Triangular Prism

Faces + Vertices = 5 + 6 = 11

Edges + 2 = 9 + 2 = 11

Faces + vertices = edges + 2

(c) i. yes ii. Yes iii. Yes

(d) True

3. (a) **Not** a polyhedron

(b) Tetrahedron

(c) Cube

(d) **Not** a polyhedron

(e) Octahedron

(f) Dodecahedron

END OF TOPIC 4

REFERENCES

- FODE Grade 9 Formal Mathematics Unit Book 4 and 5
- Oxford University Press: Oxford Mathematics; Intermediate GCSE
- Pearson 2 Units Mathematics Book 1 by S.B. Jones and K.E. Couchman
- NDOE Secondary School Mathematics 9B
- Grade 9 Mathematics, Outcome Edition
- http://forum.swarthmore.edu/sun95/suzan/hawaii.html
- http://www.mathforum.com/
- http:www.punahou.edu/acad/sanders/geometry pages/GP04Symmetry.html
- http:www.punahou.edu/acad/sanders/geometry pages/NCTMposters.html
- http:www.punahou.edu/acad/sanders/geometry pages/GP07Tessellations.html
- http://www.math.csusb.edu/faculty/susan/susan.html
- McGraw Hill: Developmental Mathematics Book 2 Fourth Edition, by: A.Thompson and E. Wrigthson; revised by S. Tisdell
- Maths for Qld 1 and 2